

Dynamic pricing efficiency with strategic retailers and consumers: an analytical analysis of short-term market interactions

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Abstract

Demand response programmes reduce peak-load consumption and could increase off-peak demand as a load-shifting effect often exists. In this research we use a three-stage game to assess the effectiveness of dynamic pricing regarding load-shifting and its economic consequences. We consider a retailer's strategic supplies on forward or real time markets, when demand is uncertain and with consumer disutility incurred from load-shedding or load-shifting. Our main results show that a retailer could internalize part of demand uncertainty by using both markets. A retailer raises the quantities committed to the forward market if energy prices or balancing costs are high. If the consumer suffers disutility, then the retailer contracts larger volumes on the forward market for peak periods and less off peak, due to a lower load-shifting effect and lower off-peak energy prices.

Keywords: Demand Response, electricity markets, load-shifting, disutility.

JEL codes: D12, L11, L22, L94, Q41.

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1. Introduction

In the transition towards a low carbon economy, deploying renewable energies (RE) and improving energy efficiency (EE) take on huge significance. By 2060, in the 2°C scenario of the International Energy Agency, RE and EE are expected, respectively, to account for 35% and 40% of reductions in greenhouse gas emissions (IEA, 2017). Investing in smart grids (Gwerder et al., 2019) is one way to make it easier to achieve these efficiency and environmental goals (Clastres, 2011; Bergaentzle et al., 2014). Moreover, retailers will be able to offer dynamic pricing to consumers, with retail prices reflecting market constraints (Chao, 2010). In this new context demand responds to prices when a dynamic tariff is introduced (Faruqui et al., 2010a; Faruqui et al., 2010b; Faruqui and wood, 2008; Faruqui and Sergici, 2010). Moreover implementing Demand Response (DR)² can yield significant economic and environmental gains (Borenstein, 2002; Borenstein et al., 2002; Borenstein, 2005; Chao, 2010; Faruqui et al., 2007; Haney et al., 2009; Hogan, 2009). These gains are linked to the decrease in peak-load prices and peak generation, and to reshaping of the demand curve to better integrate intermittent energy sources (Strbac et al., 2006; Hesser and Succar, 2011). Additional benefits could be derived from energy savings and lower bills for consumers (Dahlke and Prorok, 2019; Haney et al., 2009), or from reduced transmission and distribution investments (Strbac, 2008). However excessive load-shifting increases energy bills in off-peak periods (Rious et al., 2012) or creates additional peak periods, simply displacing the critical period (Torriti, 2012; Allcott, 2011; Spees and Lave, 2007). So such gains may be modulated by rebound and load-shifting effects³, thus increasing consumer disutility due to higher prices and efforts to save energy (Clastres and Khalfallah, 2015; Horowitz and Lave, 2014).

We use a Stackelberg-based model or bi-level programming problem (Zugno et al., 2013) to study the impact of dynamic pricing and load-shifting on a retailer's electricity supply to consumer markets. The retailer is a lead player, anticipating consumer response to the retail market. The retailer faces demand uncertainty in the day-ahead market, due to uncertain weather conditions and unpredictable consumer response to dynamic pricing. First level decisions by the retailer concern energy bought on the day-ahead market and the volume of balancing energy purchased on the real-time market. The second-level decisions relate to consumer response to dynamic pricing on the retail market. So the model can be formulated as a mathematical program with equilibrium constraints (MPEC)⁴, in which the retailer buys energy on the day-ahead market subject to real-time market equilibrium, once weather conditions are known, and to retail market equilibrium when real consumer demand is observed. Solutions of the

² The Federal Energy Regulatory Commission, defines Demand Response as “Changes in electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time, or to incentive payments designed to induce lower electricity use at times of high wholesale market prices or when system reliability is jeopardized.”

³ Overall demand could be lower, constant or higher, mainly due to rebound or load-shifting effects (Greening et al., 2000; Muratori et al., 2014).

⁴ For a formal definition of MPEC problems, see Alder et al. 2004.

overall game are found by backward induction. The mixed complementarity problem (MCP) technique is used to solve the game's sub-problems, calculating equilibrium at each stage (Gabriel and Smeers 2005). Our game can be formulated as an MCP problem since all the decision variables are non-negative, which entails complementarity between decision variables and their respective first-order conditions. Also all the constraints of our mathematical program take the form of inequalities, which define the lower or upper bounds on decision variables. The equations associated with the non-negative variables, decision variables and dual variables of inequality constraints are called complementarity conditions. A complementarity relationship between the model's constraints and their respective dual variables can then be obtained. With this approach the equilibria at each stage are defined as a set of prices and quantities which simultaneously satisfies the first order optimality conditions and complementarity conditions of the program⁵.

We show, through five propositions, that load-shifting and delaying of shifted consumption depend on the value consumers assign to their peak and off-peak consumption. In a context of high forward-prices retailers adapt their procurement strategies by contracting on the day-ahead market for off-peak hours. They anticipate substantial load-shifting and the likelihood of high balancing costs too. In so doing they minimize procurement costs and reduce retail prices for consumers.

The paper is organized as follows. Section 2 contains a review of the literature and we position our research in this context. In Section 3 we present the stochastic model and the equilibrium we obtain on the forward, real-time and retail markets. Section 4 analyses our results in terms of prices, uncertainty and consumers disutility. Section 5 concludes with the main recommendations of our study.

2. Literature review

The literature on DR has mainly focused on assessing how consumers respond to dynamic pricing. It relies largely on laboratory experiments, data analysis and econometric models (Faruqui et al. 2014), estimating short and long-term demand elasticity (Cuddington and Dagher 2015, Burke and Abayasekara 2018). Matsukawa et al. (2000) study the introduction of a time-of-use (TOU) rate in Japan. In particular their results show how consumers fitted with flexible appliances opt to receive an incentive and reduce peak consumption. Di Cosmo et al. (2014) also analyse the impact of TOU pricing on consumers by introducing various other stimuli (in-home display [IHD], monthly and bi-monthly billing). Their results show that variation in consumption does not mirror exactly variation in price, a finding that highlights the need to carry on providing frequent information to perpetuate behaviour that reduces demand. Eryilmaz et al. (2017) analyse the demand-response strategies of industrial consumers on the MISO market. They note that retail industrial consumers could participate in more DR services by optimizing the use of their flexible capacity. Frondel and Kussel (2019) use an econometric model similar to the one developed by Eryilmaz et al. (2017) to study the demand elasticity of consumers

⁵ More details on this technique are provided in Appendix 1.

receiving information on the characteristics of retail prices. They observe that demand responds to marginal prices and recommend that operators sell supply contracts based exclusively on the price per kWh. Fenrick et al. (2014) study the experimental introduction of TOU, critical peak pricing (CPP) and IHDs in Minnesota and South Dakota. They demonstrate that there is significant elasticity of substitution and consequently shifting for all (urban and rural) consumer categories.

Some of these articles address the issues of disutility and loss of welfare associated with DR and the associated services or dynamic pricing schemes. Rodrigues and Linares (2015) show that overall demand falls – in other words loads shed at peak hours are only partly shifted to off-peak hours. The slight increase in off-peak prices has only a marginal effect on consumer surplus. On the other hand the impact on collective welfare is negative following a reduction in upstream profits. Alberini et al. (2019) study demand elasticity in Ukraine, in a context of continuously high prices. Consumers must cope with inclining block rates. The authors show that consumers are aware of the pricing structure, but reduction in demand is rare and depends on the level of household equipment. Price rises have a negative impact on household surplus due to their low level of response. Woo et al. (2017) add to DR literature by using generalized Leontief demand analysis, while assuming low elasticity of substitution. They show that demand changes depending on the ratio between peak and off-peak prices, off-peak consumption displaying substantial growth with this ratio. The authors also assert that savings made thanks to DR improve welfare and compensate for the disutility incurred by consumers due to the cost of introducing DR. Simshauser and Downer (2016) study efficiency gains and inter-and intra-segment wealth transfers arising from existing flat-rate or dynamic tariffs (TOU and CPP). They show that consumers only slightly reduce overall consumption, but alter its structure by shifting usage from peak to off-peak hours. This effect is even more noticeable when network tariffs increase, which reduces the surplus enjoyed by some consumers due to a drop in cross-subsidies. Nakada et al. (2016) analyse incentives for households to invest in distributed solar-power infrastructure in Japan in order to maintain a constant level of utility with TOU pricing. To participate in DR services consumers must change their lifestyle, for otherwise their bills increase and their welfare suffers. The authors conclude that consumers who are subject to TOU and are well informed are the most likely to purchase solar power technology. In which case they participate in a DR service, reducing peak consumption without any loss of comfort, curtailed energy being supplied instead by the photovoltaic system and self-consumed. Richter and Pollitt (2018) analyse the form of contracts including smart services that consumers are prepared to pay for. The authors show positive willingness to pay (WTP) for energy-saving services (technical support, IHD, personal feedback), but negative WTP (willingness to accept) for services related to the use of consumption data and control of electricity usage. Broberg and Persson (2016) report a choice experiment estimating how willing consumers are to pay for demand-side management services. Their results show that consumers attach great importance to their comfort and to the disutility incurred from direct-load control. Furthermore, consumers are less flexible in their (peak) evening usage and load-shifting entails a cost

that must be compensated. Feuerriegel et al. (2016) show that retailers offering DR services achieve positive net present value through load-shifting. However an increase in the frequency of data-polling – and consequently in infrastructure and information technology costs – impacts profits. De Castro and Dutra (2013) focus on investment in smart grids to secure the reliability of the electricity system and set up DR. They note that investments are sub-optimal because consumers' willingness to pay for reliability does not match its true cost. The regulator must internalize the risk taken by utilities in order to restore optimal conditions for investment.

A third batch of literature studies demand response by modelling several electricity markets and the impact of transfers between agents, in particular with the introduction of dynamic pricing. Zugno et al. (2013) analyse the procurement strategy of a retailer drawing on two markets, the day-ahead market (DAM) and the real-time market (RTM), both subject to price uncertainty. The authors conclude that the retailer prefers to adopt a long position when negotiating purchases on the DAM. Demand response enables it to reduce the cost of purchases, by postponing part of peak consumption, but also to minimize imbalance costs. Welfare increases because the retailer's costs drop, with dynamic pricing, in particular real-time pricing (RTP), because it results in more DR and load management. Consumer comfort intervals are comparable. Damien et al. (2019) set out to estimate how consumers respond to DAM and RTM price signals. Their findings show that consumers are more sensitive to DAM than to RTM prices. Consumers with prior knowledge of DAM prices have more scope for adjustment. The authors also note that few consumers have an incentive to adjust their usage in real time, unless such adjustment is automated. Chao (2011) analyses the conditions of DR efficiency. The author notes that the customer baseline must be covered by a contract, between retailer and consumer, in order to make it efficient, thus overcoming any gaming incentive and double payment undermining performance. Crampes and Leautier (2015) also conclude that the optimal solution is to compensate retailers for load-management; otherwise, consumers must make a contractual commitment to baseline consumption. Chen and Kleit (2016) study the importance of calculating a customer's baseline (CBL) using data from the PJM market. They show that learning effects prompt consumers to manipulate their CBL in order to participate in more DR services, in particular through strategic use of air-conditioning. Chao (2010) notes that the introduction of real-time pricing is efficient in that it reduces cross-subsidies between peak and off-peak consumers, thus restoring efficiency in terms of the energy consumed at different times and for different price signals. Holland and Mansur (2006) report that such pricing must apply to a critical mass of consumers for it to reduce peak load but with substantial load-shifting to off-peak periods. On the other hand, many empirical experiments on RTP highlight the difficulty achieving a sufficient number of participants to really improve system efficiency (Barbose et al., 2005; Navigant Consulting Inc., 2011). Furthermore, Leautier (2014) uses an analytical approach to show that the impacts on welfare of RTP, through deployment of smart meters, is not economically efficient for all consumers. Over a critical

number of users, installing smart meters for all consumers reduces welfare, marginal gains being lower than the marginal cost of installing smart meters.

The literature modelling short or long-term electricity market interaction also focuses on the optimal strategies for generator with regard to investment, power generation and pricing behaviour (Hobbs et al. 2000; Gabriel and Smeers, 2005 and Ritz and Teirila, 2019). The strategies of retailers and consumers are generally overlooked, on the assumption that they are either passive, or are too inflexible to influence market outcome. Some research, that has considered the strategy of retailers, has examined the extent to which their interaction on forward and real-time electricity markets would affect their business. It has been demonstrated, using analytical and computational models, that retailers have an incentive to contract more energy on forward markets to secure uncertain demand (Kamat and Oren, 2004). This incentive increases when energy imbalances in real-time markets give rise to penalties (Khalfallah and Rious, 2013). However, in a context of dynamic pricing, responsive final consumers could alter such forecasts. Their strategies may distort outcomes and overall equilibria in forward, real-time and retail markets. In a context of this sort the analytical model developed here proposes, to our knowledge, an original methodology for assessing the economic impact of dynamic pricing by modelling the short-term strategies of retailers and consumers.

Our paper adds to the literature on modelling three-stage stochastic games which focuses on optimal strategies for consumers and retailers in the foreseeable context of dynamic pricing. We model the following three market stages: day-ahead, real-time, and retail market. Day-ahead market decisions are made with an uncertain expectation of future demand, so we assume a closed-loop information structure to simulate interactions between players' decisions. A second originality of this work is that it uses an analytical approach to solve the game, hence a more robust, widely applicable solution to assist decisions by policy-makers. This contrasts with the literature in which only numerical applications are used to find solutions based on parameter specifications (Saguan and Meeus, 2014). To capture consumer disutility, our paper also assumes a coefficient. So our research could contribute to understanding consumer behaviour, by focusing on their consumption profile. A strong disutility parameter could indicate great interest in peak consumption, due to a lack of flexibility or a preference for peak-load energy usages.

3. Model

We introduce a three-period stochastic model to study how dynamic pricing affects retail decisions by consumers and short-term outcomes on electricity markets as a whole. The basic idea behind this model is that when moving from a regulated to a dynamic pricing scheme, end consumers should be encouraged to adopt more energy-efficient patterns of consumption, either by reducing overall usage or at least delaying it. However, the economic impact of such energy efficiency in terms of social welfare has rarely been explored. Consumers choose between costly energy and the disutility of reducing

consumption, or load-shifting. On the other hand, the retailer must cope with real-time demand that is not only uncertain but also uncontrollable, consumers now being responsive. So the retailer must change their strategy on short-term markets.

3.1. Model assumptions

The model considers the decisions taken by energy retailers⁶ in day-ahead and real-time markets, and decisions by consumers at two typical times of the day: off-peak and peak hours⁷. We consider uncertainty affecting future demand when the retailer decides to purchase energy on the forward market. Such uncertainty is represented by a finite set of scenarios.

The model consists of three periods (see Figure 1). We assume that, at each period, the players observe all the actions of previous periods. They base current decisions on that information and on their “correct” rational expectation of the behaviour of all the other players in the current period and on the outcomes of subsequent periods.

We shall now explain the model backward. The third and last period (the retail market) represents the consumption period⁸. The period is divided into two sub-periods, off-peak and peak. To analyse how consumers respond to dynamic pricing, we assume a benchmark case in which power is supplied at a regulated, flat price. We then analyse how consumers adapt their choice under dynamic pricing. In the benchmark case, consumer demand, when known, is inelastic for the two sub-periods. Under dynamic pricing, an electricity price function is offered to consumers. They decide on the volume required to meet their real needs, but they can now reduce or shift consumption from the peak to the off-peak period to avoid paying excessive prices. With the roll-out of smart technologies, retailers can offer consumers a contract based on dynamic pricing (such as CPP or RTP). In this way, retailers charge consumers a price that reflects both the degree of competition on the retail market and equilibria on wholesale energy markets. Consumers must adapt their demand to suit these new pricing schemes, to avoid higher energy bills and some disutility entailed by dynamic pricing. However, we assume that reducing or shifting peak consumption would certainly create disutility⁹. Consumers must spend time, frequently monitoring

⁶ Only one retailer is modelled in this study. The possibility that consumers switch to a different retailer is not considered. This assumption can be justified in three ways. Firstly, the process of switching retailer is rather slow compared to the optimization horizon considered here. It is true that competition between retailers should have an impact on the dynamic-pricing outcome and on retail prices in general, but this impact should be low. It is widely argued that the final electricity price mainly reflects player interaction in upstream markets, so the outcomes of day-ahead and real-time markets are sufficient to signal the electricity price paid by end-consumers. Finally, the paper analyses the welfare efficiency of dynamic pricing by focusing on interaction between the retailer and flexible consumers in their respective markets. Competition between retailers can be disregarded without trivializing the scope of the study.

⁷ As in Zugno et al. (2013) we make no allowance for network-access tariffs. Positing a risk-neutral system operator, as in Simshauser and Downer (2016), leads to higher transmission tariffs under DR. This increase may be captured by the resale factor γ which increases sale prices in the retail market.

⁸ We assume one representative consumer and that consumers are homogeneous, with the same demand (Leautier, 2014) and disutility functions.

⁹ For detailed consideration of the causes of disutility following the introduction of DR or dynamic pricing, see Nakada et al. (2016).

signals from the retailer, smart appliances and other sources of information to adjust their usage in line with market conditions (Nakada et al., 2016). These changes take time and consequently incur disutility. To maximize utility consumers determine how much they consume at the retailer's price and for each sub-period. They then decide whether to shift or shed loads. The volumes in play will depend on wholesale prices and retailer costs.

In the second period, the real-time market, effective consumer demand is known, which could be normal or extreme with regard to prior expectations on the day-ahead market. For simplicity's sake we assume that at this stage the retailer is a passive player¹⁰. It has an obligation to balance its demand in line with the consumption it really serves. On the other hand, the retailer faces two possible market situations. Either it has bought more energy from the day-ahead market than justified by the demand it actually serves, or it has bought less. The real-time clearing price is then defined by considering that retailers can be penalized beyond the marginal price of electricity in real-time if an imbalance is observed¹¹. The argument underpinning this design is that the retailer has less incentive to rely on real-time market to supply the load demanded by its end consumers. The penalty is generally calculated as a function of the type of retailer imbalance (positive, due to surplus forward volume in relation to load; or negative, volume shortfall in relation to load). To simplify our model, the penalty, applied to marginal price, is computed as follows: the negative (or positive) imbalances prices are computed explicitly by multiplying (or dividing) the marginal price of electricity in the real-time market by a constant¹².

Lastly, in the first period (the day-ahead market), we formulate a stochastic problem in order to optimize the retailer's day-ahead commitments¹³. The formulation is a MPEC in which equilibrium constraints are the consumer's best reply on volume in the subsequent retail market, the retailer's real-time balancing volume, the real-time price paid by the retailer, and the retail price paid by the consumer. The retailer buys a day-ahead volume knowing how the consumers will optimally respond in the subsequent real-time stage, for each realization of expected real-time market demand. At this stage, the retailer faces uncertainty as to the level of demand it must serve in the subsequent real-time step.

¹⁰ This assumption is realistic since in real-time markets, with or without balancing schemes, the retailers are under a greater obligation to balance their specific demand rather than to act strategically and try to manipulate prices. Such behavior is more likely in forward markets.

¹¹ For instance, a balancing market is applied in France. Imbalance prices reflect the costs of system balancing in real time. Imbalances are priced regarding the price of activated energy (positive or negative). A penalty is then added to the average offered price and paid by market participants that do not balance their schedules ex-ante in the forward markets (RTE, 2020). In Belgium, imbalance prices are however calculated under marginal pricing scheme. See (ENTSO-E, 2017) for an overview of balancing markets schemes applied un Europe.

¹² Since there is only one retailer in our model, its imbalance corresponds to the system imbalance.

¹³ We consider the day-ahead market as the common forward market where market players take decisions subject to uncertain demand. However, the study could be extended to include more forward contracts with different time maturities.

\bar{Q}_i^{RT}	maximal electricity demand, expected in next period i
\bar{Q}^{RT}	maximal total expected electricity demand
γ	resale factor ¹⁴
c	retailers' delivery cost
α	penalty factor in real-time
m, n	intercept and slope of marginal cost function of marginal unit on the day-ahead market $Q_i^{DA} \mapsto MC_i^{DA} = m + n * Q_i^{DA}$
β_i	(dis)utility factor, given the decision to delay consumption in period i
μ_i	binary parameter of load-shifting equal to 1 in peak period (disutility) and -1 in off-peak (utility).
\bar{P}_i^R	regulated retail price before dynamic pricing, in period i

3.3. Model Formulations

Third period (retail market stage)

We start by formulating the third-period problem. To analyse consumer response to dynamic pricing, we assume a benchmark case in which power is supplied at a regulated, flat rate \bar{P}_i^R . Nature determines the state of the world. For a given state w , consumer demand, when known, is inelastic and can be expressed by \bar{Q}_i^{RT} and \bar{Q}_h^{RT} respectively for off-peak and peak periods. With dynamic pricing, an electricity price function is offered to consumers. They decide on volume ($Q_{i,w}^R$) depending on their real needs, but they can now reduce or shift consumption to avoid paying possibly excessive prices ($P_{i,w}^R$). However, reducing or shifting consumption at peak hours certainly entails disutility. Consumers maximize their total utility to determine which volumes they consume at the retailer's price. Volumes will depend on wholesale prices and the retailer's costs. We define the consumer utility function in a period i as:

$$\max_{Q_{i,w}^R} \Delta S_{i,w} = (\bar{P}_i^R - P_{i,w}^R) \cdot Q_{i,w}^R - \mu_i \beta_i \cdot (\bar{Q}_i^{RT} - Q_{i,w}^R)^2 \quad (1)$$

Subject to,

$$\sum_i Q_{i,w}^R \leq \bar{Q}^{RT} \quad (\lambda_1) \quad (2)$$

¹⁴ The resale factor γ may integrate the various costs related to supply and competitive mark-up (retail margin).

$$Q_{h,w}^R \leq \bar{Q}_h^{RT} \quad (\lambda_2) \quad (3)$$

$$Q_{l,w}^R \geq \bar{Q}_l^{RT} \quad (\lambda_3) \quad (4)$$

$$Q_{i,w}^R \geq 0 \quad (5)$$

Where,

$\lambda_i \geq 0$ Dual variables of the constraints

$$P_{i,w}^R = \gamma \cdot P_i^{DA} \text{ and } \gamma > 1 \quad (6)$$

The consumer utility function (1) is defined as the variation in consumer surplus when moving from regulated to dynamic pricing. The first term captures the price effect of dynamic pricing as consumers now pay the real-time price ($P_{i,w}^R$). The second term shows the volume effect of dynamic pricing. It assumes that consumers incur an opportunity cost from reducing or shifting peak consumption ($\bar{Q}_h^{RT} > Q_{h,w}^R$). β_h is the disutility factor at peak hours, whereas off-peak consumption cannot be lower than the consumption benchmark ($Q_{l,w}^R \geq \bar{Q}_l^{RT}$), so shifting the load to off-peak hours, would generate an opportunity gain for consumers (β_l) without fully compensating the disutility of load-shifting ($\beta_l < \beta_h$)¹⁵. Stated differently, reducing energy usage or load-shifting is only possible at peak hours. During off-peak hours, consumers are not really affected by dynamic pricing, prices are obviously attractive and they may even increase their consumption to compensate for reduced peak demand. We assume a non-linear increasing function for the opportunity cost or gain of load-shifting, $(\beta_i \cdot (\bar{Q}_i^{RT} - Q_{i,w}^R)^2)$, to allow for the increasing marginal impact of load-shifting on consumer welfare, regardless of their consumption profile.

Dynamic pricing schemes are designed to reduce overall consumption. So we assume a set of constraints (2-5) expressing the fact that additional peak consumption can only decrease, whereas off-peak will increase because of possible load-shifting.

The assumptions for the previously specified model ensure that (1-6) is a convex programming problem, which implies that first order conditions are sufficient for optimality (Gabriel and Smeers, 2005). So, to solve the period-3 problem, we can just formulate a MCP program that can be solved as discussed in Appendix 1.

¹⁵ Off-course, the decrease in consumers' welfare because of load-shedding in peak period could not be totally compensated by the load-shifting to off-peak period.

Second period (real-time market stage)

In real time effective consumer demand is known. The retailer faces two market situations: either it has bought more energy on the day-ahead market than the demand it has really served; or it has bought less. The real-time clearing price can thus be defined as:

$$P_{i,w}^{RT} = \begin{cases} \alpha \cdot P_i^{DA} & \text{if } Q_i^{DA} < Q_{i,w}^{*R} \\ \frac{1}{\alpha} \cdot P_i^{DA} & \text{if } Q_i^{DA} > Q_{i,w}^{*R} \end{cases} \quad (7)$$

The retailer is penalized by paying more than the market price in the event of negative system imbalance ($Q_i^{DA} < Q_{i,w}^{*R}$) and otherwise by receiving a lower price. It has an obligation to balance its demand with regard to the real consumption response function, as determined in the previous program. The retailer's profit can thus be determined as follows:

$$g_{i,w}^{RT} = \sum_i (P_{i,w}^R - c) Q_{i,w}^{*R} - \sum_i P_{i,w}^{RT} \cdot Q_{i,w}^{*RT} \quad (8)$$

where,

$$Q_{i,w}^{*RT} = Q_{i,w}^{*R} - Q_i^{DA} \quad (9)$$

is the real-time quantity bought or sold by the retailer to balance the demand it serves.

First period (day-ahead market stage)

In the day-ahead market, the retailer schedules its load before the operating day. It faces uncertainty as to the level of demand it will serve in the subsequent real-time step. Considering these uncertainties, the retailer chooses the quantities Q_i^{DA} it needs to buy for each consumption sub-period i by maximizing its expected profit with regard to its purchase strategies. To do so, we formulate a stochastic problem in order to determine the optimal day-ahead contracts for the retailer. This formulation takes the form of a MPEC problem in which the equilibrium constraints are integrated in the model below: real consumer demand in period i and demand state w , $Q_{i,w}^{*R}$, the retailer's real-time balancing volume, period i and demand state w , $Q_{i,w}^{*RT}$ and the respective real-time price and consumer retail price, $P_{i,w}^{RT}$ and $P_{i,w}^R$. In other words, in this stage, the retailer buys the volume Q_i^{DA} knowing how the consumers will optimally respond in the subsequent real-time stage, for each realization of the expected real-time market demand.

Basically, maximizing MPEC problems are constrained by a non-concave region, so it is difficult to simply write down the necessary first-order conditions and aggregate them into a large problem to be solved directly. Non-concavity would generally lead to multiple local optima or the absence of equilibrium (Sauma and Oren, 2006). In our case the first period's MPEC maximization problem is rearranged and defined as a concave maximization function. The retailer, which leads our bi-level programming problem, maximizes a concave expected profit, integrating the optimality conditions for the second and third periods. The optimization problem thus becomes analytically tractable (Hobbs et

al., 2000). Appendix 2 provides details of our methodology to explain and justify the resolution of the model below and asserts its tractability.

The overall retailer optimization problem is described as follows:

$$\max_{Q_i^{DA}} [E_w(Profit) = - \sum_i P_i^{DA} * Q_i^{DA} + E_w(\sum_{w,i} g_{i,w}^{RT})] \quad (10)$$

Subject to,

$$Q_i^{DA} \leq \bar{Q}_i^{RT} \quad (11)$$

$$Q_i^{DA} \geq 0 \quad (12)$$

and all optimally conditions of period 2 and 3 problems.

Where,

$P_1^{DA} = MC_1^{DA}$ the day-ahead electricity price which corresponds to the marginal cost function of the marginal producer.

The first term in (10) shows the cost of the energy bought by the retailer in the day-ahead market. The day-ahead price is assumed to correspond to the marginal cost of generators. The strategic behaviour of generators is ignored here and a competitive price is assumed¹⁶. The second term shows the expected outcome for the retailer from buying or selling energy in the subsequent real-time steps and selling energy to consumers in the retail market. The retailer faces uncertain real-time demand. So it must buy energy on the forward market subject to uncertain demand, only ultimately observed in real-time. We introduce a random variable w that indicates possible demand realization in real-time and corresponds to a finite set of scenarios.

As demonstrated in Appendix 2, optimizing the mathematical program (1-12), in which optimal best-reply function in real-time and retail markets are integrated, yields the following results:

$$\left\{ \begin{array}{l} Q_h^{*DA} = \frac{V.T+2.U.L}{V^2-4.U^2} \\ Q_l^{*DA} = \frac{V.L+2.U.T}{V^2-4.U^2} \end{array} \right. \quad (13)$$

Where U, V, T and L are expressions that depend on demand uncertainty and parameters of price and demand functions (Appendix 2). Considering this equilibrium, we may now compute consumer demand on the retail market, the volumes committed to the real-time market and load-shifting.

¹⁶ Dynamic pricing should modify patterns of consumption and energy trades in the retail market. The model here looks mainly at interaction between consumers and load-serving bodies rather than generator strategies in upstream markets.

4. Theoretical results

To analyse decisions taken by retailers on the day-ahead market and then overall game decisions, the complex solutions shown in Appendix 2 are rearranged to yield more tractable and subtle results. Analysis of sensitivity to the model's main market parameters is undertaken, by modifying some of the main parameters, but leaving all the others unchanged. Firstly we consider the parameters β , which must strongly impact consumer choices, and hence retailer strategies, once dynamic pricing is applied. Then we analyse the parameter γ , which captures the degree of competition faced by consumers in the retail market and the extent to which consumer strategies in this market could be altered by the simultaneous application of dynamic pricing and balancing mechanisms. We next consider the parameter α , which indicates how sensitive the efficiency of real-time pricing could be to the intensity of the balancing mechanism, with $\alpha > 1$. We conclude by analysing the results regarding the parameter n which expresses a price signal from the market merit order and consequently the specific technology mix of a given system. Sensitivity analysis is illustrated by the figures below, which reflect the analytical results discussed in Appendix 2.

Our parameters are intuitively correlated with one another. The penalty for correcting imbalances " α " is positively correlated with the slope of the marginal cost of the last unit traded on the day-ahead market " n ". When the day-ahead market is under tension it will cost more to balance supply and demand than under less tense conditions, given the energy available on the balancing market. There is also a positive correlation between disutility " $\beta_h - \beta_l$ " and parameter " γ ", allowing for the dynamic pricing it entails and competitive mark-up. Consumers are increasingly likely to trim usage when the retail price is high, with a positive impact on their disutility. The same reasoning applies to explain the intuitively positive correlation between " n " and " $\beta_h - \beta_l$ ". Making allowance for these intuitive correlations amplifies our results, which remain valid.

4.1. Dynamic pricing and consumption efficiency

Proposition 1: Dynamic pricing encourages load-shifting to off-peak periods. Load-shifting decreases with consumer disutility.

Optimizing the problem facing the consumer, as demonstrated in Appendix 1, yields the following results:

$$Q_{h,w}^{*R} = \bar{Q}_h^{RT} - \frac{\bar{\Delta}P_w^R}{2\bar{\Delta}\beta} \quad (15) \quad \text{and} \quad Q_{l,w}^{*R} = \bar{Q}_l^{RT} + \frac{\bar{\Delta}P_w^R}{2\bar{\Delta}\beta} \quad (16).$$

Where,

$$\bar{\Delta}\beta = \beta_h - \beta_l \quad \text{Net load-shifting disutility}$$

$$\bar{\Delta}P_w^R = P_h^R - P_l^R \quad \text{Price difference between peak and off-peak periods}$$

The solutions in (15) and (16) show that consumers would shift their consumption from peak to off-peak hours¹⁷, regardless of climatic conditions, w . The volume shifted, $\frac{\bar{\Delta}P_w^R}{2\bar{\Delta}\beta}$, depends on peak and off-peak retail prices, load-shifting disutility and corresponding compensation, and the difference between the two. The greater the price differential, the more attractive it is for consumers to shelve peak consumption and shift it to off peak period. The analysis of partial derivatives confirms this result. Indeed, $\frac{\partial Q_{h,w}^{*R}}{\partial(\bar{\Delta}P_w^R)} < 0$ and $\frac{\partial Q_{l,w}^{*R}}{\partial(\bar{\Delta}P_w^R)} > 0$. However this effect is greatly diminished by the intensity of load-shifting disutility, $\frac{1}{\Delta\beta}$. Partial derivatives show that peak demand decreases with disutility, $\frac{\partial Q_{h,w}^{*R}}{\partial(\Delta\beta)} < 0$, rather than off-peak one increases, $\frac{\partial Q_{l,w}^{*R}}{\partial(\Delta\beta)} > 0$. For instance, if consumers are less sensitive to environmental concerns, or places a high value on comfort to satisfy their energy needs, they will attach more value to the impact of shedding energy on their consumption habits and incur greater disutility from load-shifting, i.e. $\beta_h \gg \beta_l$. Conversely, load-shifting is more likely to occur when consumers incur a lower opportunity cost from load-shifting or register a significant opportunity gain from off-peak usage, i.e. $\beta_h \sim \beta_l$. Using partial derivatives, we could easily show that the disutility effect is greater than the price differential effect if $\bar{\Delta}\beta < \frac{\bar{\Delta}P_w^R}{2}$.

Turning now to the retail price, $P_{i,w}^R$. It is defined as a function of the day-ahead price and depends on Q_i^{DA} . If we replace the retail price in the solutions to Proposition 1, we obtain the following optimal load-shifting ($Q_{rep,w}^*$), as a function of the retailer's day-ahead commitments:

$$Q_{rep,w}^{*RT} = \frac{\gamma \cdot n}{2\bar{\Delta}\beta} \cdot (Q_h^{DA} - Q_l^{DA}) \quad (17)$$

This last result is a reminder of how much load-shifting increases when the compensation gain from load-shifting is significant and $\bar{\Delta}\beta$ is low, but it also highlights the impact of the resale factor γ and energy supply elasticity n . These two parameters can be interpreted as the market price signals to consumers: high levels warn consumers that energy is costly, encouraging them to shift their load. Lastly, as we assume a linear increase in the energy price function, we observe that, as the forward-volume differential rises, the higher peak prices climb, the greater the incentive is for consumers to shed demand.

¹⁷ As demonstrated in Appendix 1, other equilibrium configurations, such as only reducing energy consumption at peak hours or reducing energy consumption more or less than load-shifting, are not possible. The optimization problem, being linear, only provides solutions at boundary points.

4.2. Market interactions

Proposition 2: *If the consumer has a lower preference for peak consumption, then the retailer can internalize the load-shifting effect, contracting more off-peak volumes on the forward market.*

Figure 2 below illustrates Proposition 2 that agrees with the results of Zugno et al. (2013). Consumers incur an opportunity cost β_h when obliged to reduce peak usage. Their preference for off-peak usage β_l could be equal to or less than β_h . So, if consumers have no preference regarding the consumption period, i.e. $\lim_{\beta_l \rightarrow \beta_h} \bar{\Delta}\beta = 0$, the retailer will anticipate that consumers will shift part of their peak demand to an off-peak period, which offers lower prices. The retailer will consequently contract greater volumes on the forward market to serve off-peak hours and lower volumes for peak periods. However, off-peak prices increase with off-peak demand. Consumers have an incentive to shift demand as long as the difference between peak and off-peak prices compensates for the disutility of reducing peak consumption. Moreover, as the literature has shown it (Faruqui et al., 2010a; Faruqui and Wood, 2008), consumers only shift part of their peak demand, running some electrical appliances at other times being impossible. When $\lim_{\beta_l \rightarrow \beta_h} \bar{\Delta}\beta$ is positive, consumers face greater disutility (surplus gains in off-peak hours do not compensate for surplus losses in peak hours). So the incentives to shift uses diminish. With consumers cutting back load-shifting, the retailer would contract much larger volumes for peak rather off-peak hours. Load-shifting can create a second form of uncertainty for the retailer. Its response might be to adjust forward contracts to allow for consumer behaviour and typology.

Parameters β_l could also represent consumer sensitivity to environmental factors. If β_l is close to β_h , consumers attach importance to off-peak consumption, reducing the environmental impact of peak generation (Dahlke and Prorok, 2019; Bergaentzle et al., 2014). In this way, consumers will shift as much demand as possible from peak to off-peak hours, in turn affecting the retailer's forward-market procurement strategy. On the other hand, if consumers attach little value to the environment (higher values of β_h), they will not shift a large share of usages because peak-hour consumption creates greater utility than load-shifting.

Figure 2 shows that Q_h^{*DA} could converge towards Q_l^{*DA} for low values of $\bar{\Delta}\beta$. This case illustrates the equality of peak and off-peak supplies committed on the forward market. On the left side of the graph, load-shifting is significant; $\bar{\Delta}\beta$ driven by parameters n and γ . On the right side of the graph, as $\bar{\Delta}\beta$ increases, the retailer begins to contract greater volumes for peak hours, in anticipation of a lower load-shifting effect. This analysis also shows the importance of studying consumer behaviour and typology to understand their preferences and foreseeable load-profile.

The uncertainty of demand is due to load-shifting strategies but also to its own characteristics (in our two demand scenarios, it could be normal or extreme). Thus, probabilities w also affects the volumes committed to the forward market in a very intuitive way. If the likelihood of facing extreme demand

increases, the volume committed on the forward market will also increase. As we shall see below, this effect is heightened by the level of penalties on the real-time market.

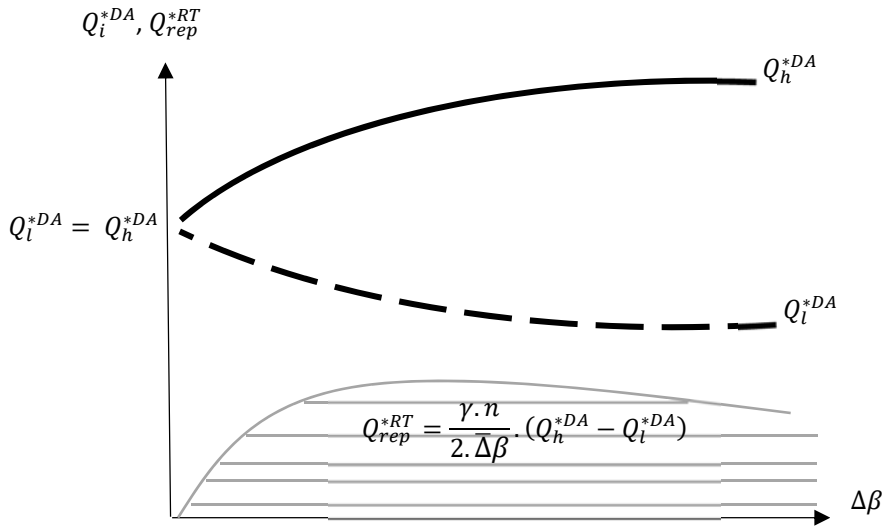


Figure 2. Impact of consumer disutility $\Delta\beta$ on forward commitments

Proposition 3: Dynamic but high pricing still encourages load-shedding whereas any imbalance costs in real-time are easily transferred to consumers.

Figure 3 below shows a linear increase in volumes bought by the retailer on the day-ahead market if γ increases. If the retail price increases in relation to wholesale market prices, due to lack of competition or retail market power, the retailer will choose to buy more energy than usual on the forward market. The retailer makes a trade-off between day-ahead market purchases, which influence the day-ahead price and hence the retail price, and real-time market purchases, on which it is only subject to imbalance costs. Any risk of facing penalties in the case of positive imbalances is passed on to consumers, thanks to additional retail-market revenues, through higher γ .

It also shows that the expectation of consumer load-shifting from peak to off-peak hours would increase with γ , which is a predictable result given that consumers will face higher tariffs, further confirming the merits of load-shifting. Moreover, the shaded area in Figure 3 shows that, whereas load-shifting potential is constant ($Q_h^{*DA} - Q_l^{*DA}$ is constant), insignificant load-shifting is expected when γ is low, reaching a high point with extreme values of γ . This means that the expected response of consumers only slightly affects the retailer's optimal trade-off between the day-ahead and real-time markets. Higher retail tariffs would be sufficient for the retailer to cover any price volatility in real-time.

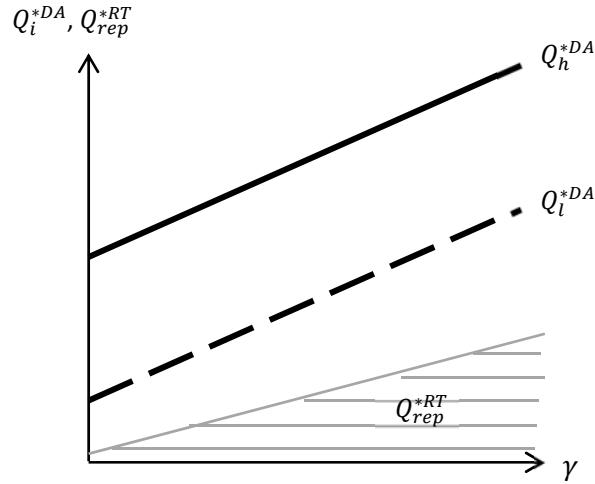


Figure 3. Impact of γ on decisions by the retailer and by consumers

4.3. Balancing design

Proposition 4: *Real-time penalties encourage the retailer to secure demand on the day-ahead market but have no impact on the efficiency of dynamic pricing.*

Penalizing real-time imbalances should encourage market players to contract sufficient energy on forward markets. The strategy of market players responding to the severity of real-time imbalance penalties could change with the switch to dynamic pricing. For instance, with increasingly price-sensitive consumer demand, retailers may face greater uncertain real demand and a higher likelihood of real-time imbalances. Figure 4 below plots the optimal day-ahead volumes of retailers as the penalty factor α increases. Two main findings are apparent. Firstly, when switching from no penalties, $\alpha = 1$, to a penalty scheme, $\alpha > 1$, the retailer has an incentive to make a higher energy commitment on the day-ahead market.

When comparing this result with the previous one, we conclude that the increase in forward volumes, as α rises, is less proportional to an increase in γ . Whereas the increase in γ positively impacts the retail price – yielding higher real-time profits for the retailer and scope for covering any penalty incurred by a positive imbalance in real-time – an increase in α is not passed onto retail price. The retailer will respond moderately to the expectation of higher imbalance prices than previously. However, this is the explanation on why consumer decisions on load-shifting are not affected by the level of α . Our model assumes that retail prices only depend on day-ahead prices, and are consequently not affected by the level of penalties, at least in the short run. This assumption is realistic. In theory, a competitive retail price should reflect two components: the energy-supply cost, in other words the merit-order function, and short-term demand elasticity, in order to allow for consumer preferences and weather conditions. The linear relationship between day-ahead and retail prices reflects these constraints. However, real-

time imbalance prices are the sole responsibility of retailers and/or generators and they must recover their full costs.

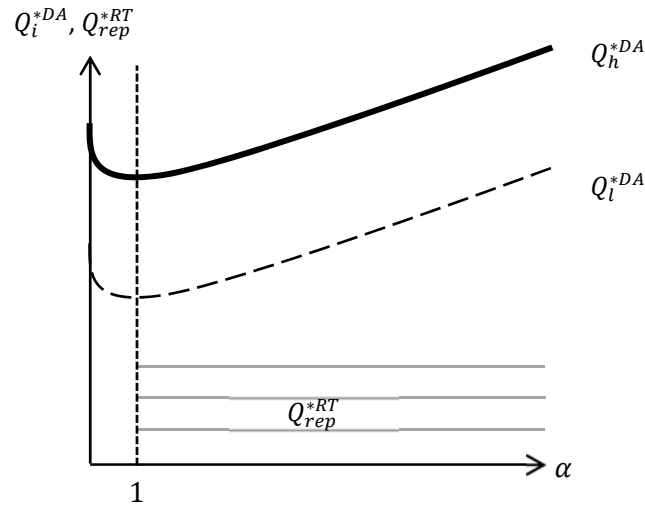


Figure 4. Impact of α on decisions by the retailer and by consumers

4.4. Markets competitiveness

Proposition 5: *With costly energy mix or low market competitiveness, the retailer significantly increases its forward volumes and transfers the cost of short-term price distortion to end consumers.*

This proposition is based on the analysis of the sensitivity of equilibria to parameter n in the marginal cost function. This parameter stands for the level of energy prices in each market and supply-function elasticity on forward markets. Q_h^{*DA} and Q_l^{*DA} are increasing functions of n . An increase in n entails higher energy prices on all markets we study and with a high level of supply elasticity. This relationship prompts several intuitions. Firstly, when energy is cheap (low values of n) the retailer will only buy small volumes on the forward market, in line with uncertain real-time demand and load-shifting effect it expects. The retailer internalizes part of the demand uncertainty, in order to balance its position on the real-time market. Such strategies are possible because of low energy prices and remain valid for a wide range of penalties on the real-time market, on condition that marginal revenue from sales compensates for higher penalties. Nevertheless, when the energy price rises (the gradient of the supply function n is higher), it becomes more expensive to reduce the *ex-ante* value of demand uncertainty. Therefore, the retailer would rather increase the volume committed on the forward market. Secondly, high energy prices increase the incentive for consumers to shift a larger share of demand, parameter n impacting positively on energy load-shedding. Thirdly, the difference between Q_h^{*DA} and Q_l^{*DA} increases with n (Figure 5). As energy prices are higher at peak rather than off-peak hours, the retailer has an incentive to book a large share of its supplies at peak hours, because of the additional balancing costs. So the

retailer could internalize some of the demand uncertainty at off-peak hours, when energy prices are lower, and so it could face the imbalance costs in real-time without a huge increase in supply costs.

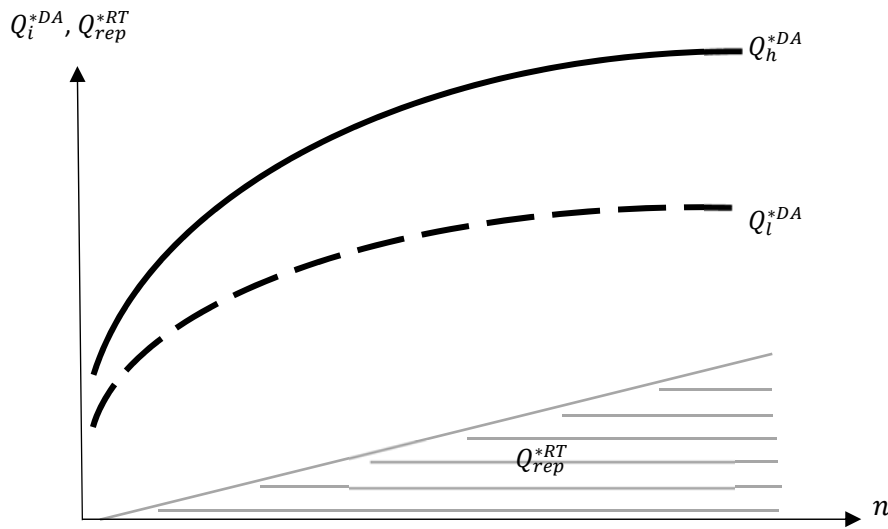


Figure 5. Impact of generating costs n on forward commitments and load-shifting

5. Conclusion

Deployment of dynamic pricing offers consumers the opportunity to respond to market conditions. Thus, they may adapt consumption in line with market and retail prices. They can shift part of their peak demand to cheaper off-peak periods. This behaviour increases uncertainty for the retailer as to market demand. Indeed, it must cope with two kinds of uncertainty: the forecast level of demand may be normal or extreme; and consumers may shed part of their load.

Using a dynamic stochastic model, we show that the supply strategies a retailer adopts on the forward market affect final demand through load-shifting, in so far as such strategies change the market price. The retailer may contract larger volumes on the forward market if generating costs or penalties on the real-time market are high enough. Its prime objective in such cases is to reduce supply costs in order to avoid bigger shifts in retail demand. Moreover, as balancing costs are not passed on to consumers, the retailer must reduce them in the event of high energy prices (on the forward and retail markets, which induces lower peak demand) or higher penalties. The load-shifting effect also depends on the disutility consumers incur from shifting consumption from peak to off-peak hours. If they perfectly assess consumption during the two periods, or if, for instance, they are environmentally aware, consumers will shift as much demand as they can. To allow for the load-shifting effect, the retailer must therefore contract higher off-peak volumes on the forward market and lower peak energy volumes. Lastly, the severity of penalties has no impact on load-shedding, balancing costs being borne by the retailer.

The retailer, facing uncertainty as to demand, protects itself on the forward market to minimize the impact of load-management and shifting on costs. This conclusion contributes to the debate on

contractualizing baseline consumption so that consumers or demand-response providers shoulder part of the risk that load management poses for balancing. Knowledge of consumer preferences regarding electricity usages is a key factor in achieving an optimal balance between supply and demand in the face of flexible consumption. On the one hand, such knowledge makes it possible to target consumers with a high DR potential; on the other hand, it minimizes the impact on their utility of changes in their behaviour. Experiments and pilot schemes designed to study the impact of dynamic pricing and DR schemes are needed to optimize the positive outcomes of such policies. Network operators will undoubtedly play an increasing part in these new, flexible-demand configurations. Ultimately, they will be able to issue dynamic price signals based on network constraints. This possibility will give rise to further research on the relationship between network operators, retailers and consumers, or demand-response providers in order to share out the risk of balancing.

Appendix 1: Use of the MCP method to find the equilibrium at the consumer stage

At each period i and demand state w , the consumer maximizes its utility (4) subject to constraints (5-9). The decision variable is the consumed volume $Q_{i,w}^R$. To state the model as a MCP problem we need to reformulate the consumer optimization problem as follows:

To calculate the optimality conditions of each program, we first define the Lagrangian function of the corresponding optimization problem $L_{i,w}$:

$$L_{i,w} = -\left(\bar{P}_i^R - P_{i,w}^R\right) \cdot Q_{i,w}^R + \beta_i \cdot \left(\bar{Q}_i^{RT} - Q_{i,w}^R\right)^2 - \lambda_1 \cdot \left(\bar{Q}^{RT} - \sum_i Q_{i,w}^R\right) - \lambda_2 \cdot \left(\bar{Q}_h^{RT} - Q_{h,w}^R\right) - \lambda_3 \cdot \left(Q_{i,w}^R - \bar{Q}_i^{RT}\right) \quad (18)$$

Then, we calculate the gradient of the Lagrangian function with respect to the two decision variables $Q_{i,w}^R$:

$$\frac{dL_{i,w}}{dQ_{h,w}^R} = -\left(\bar{P}_h^R - P_{h,w}^R\right) - 2 \cdot \beta_h \cdot \bar{Q}_h^{RT} + 2 \cdot \beta_h \cdot Q_{h,w}^R + \lambda_1 + \lambda_2 \quad (19)$$

$$\frac{dL_{i,w}}{dQ_{i,w}^R} = -\left(\bar{P}_i^R - P_{i,w}^R\right) + 2 \cdot \beta_i \cdot \bar{Q}_i^{RT} - 2 \cdot \beta_i \cdot Q_{i,w}^R + \lambda_1 - \lambda_3 \quad (20)$$

Optimality conditions of the consumer are:

$$\left(\begin{array}{l} \frac{dL_{i,w}}{dQ_{h,w}^R} \geq 0 ; Q_{h,w}^R \geq 0 \text{ and } \frac{dL_{i,w}}{dQ_{h,w}^R} \cdot Q_{h,w}^R = 0 \\ \frac{dL_{i,w}}{dQ_{i,w}^R} \geq 0 ; Q_{i,w}^R \geq 0 \text{ and } \frac{dL_{i,w}}{dQ_{i,w}^R} \cdot Q_{i,w}^R = 0 \\ (\bar{Q}^{RT} - \sum_i Q_{i,w}^R) \cdot \lambda_1 = 0 \\ (\bar{Q}_h^{RT} - Q_{h,w}^R) \cdot \lambda_2 = 0 \\ (Q_{i,w}^R - \bar{Q}_i^{RT}) \cdot \lambda_3 = 0 \\ Q_{i,w}^R \geq \bar{Q}_i^{RT} \\ Q_{h,w}^R \leq \bar{Q}_h^{RT} \\ \sum_i Q_{i,w}^R \leq \bar{Q}^{RT} \\ \lambda_j \geq 0 \end{array} \right)$$

This set of equations consists of the first-order conditions multiplied by their corresponding decision variables and the inequality constraints multiplied by their corresponding dual variables, all equal to zero; next the inequality constraints themselves; and finally, the explicit statement of the dual variables.

Grouping all these conditions together leads to an MCP problem. Equations (15-16) are therefore the solutions to this MCP problem.

Existence and uniqueness of the solution:

Given that the maximization objective function is concave and continuously differentiable, the KKT conditions presented above are necessary and sufficient for optimality since the feasible region is polyhedral (Bazara et al., 1993).

Appendix 2: A mathematical program with equilibrium constraints to find the solutions to the overall game

The retailer decides its day-ahead volumes by maximizing its expected profits in the three market stages, where consumer best-reply functions are integrated. The stochastic MPEC model in Section 3.3 is detailed as following:

$$\max_{Q_i^{DA}} E_w(Profit) = E_w \left(\sum_{i,w} \left((P_{i,w}^R - c) \cdot Q_{i,w}^{*R} - P_{i,w}^{RT} \cdot Q_{i,w}^{*RT} \right) \right) - \sum_i P_i^{DA} * Q_i^{DA} \quad (21)$$

Subject to,

$$Q_i^{DA} \leq \bar{Q}_i^{RT} \quad (\lambda_4) \quad (22)$$

$$Q_i^{DA} \geq 0 \quad (23)$$

And equilibrium constraints are (third stage MCP program):

$$Q_{h,w}^{*R} = \bar{Q}_h^{RT} - \frac{\bar{\Delta}P_w^R}{2 \cdot \bar{\Delta}\beta} \quad (24)$$

$$Q_{l,w}^{*R} = \bar{Q}_l^{RT} + \frac{\bar{\Delta}P_w^R}{2 \cdot \bar{\Delta}\beta} \quad (25)$$

$$Q_{rep,w}^{*RT} = \frac{\gamma \cdot n}{2 \cdot \bar{\Delta}\beta} \cdot (Q_h^{DA} - Q_l^{DA}) \quad (26)$$

$$Q_{l,w}^{*RT} = Q_{l,w}^{*R} - Q_l^{DA} \quad (27)$$

As for optimizing the consumer sub-problem, we calculate the gradient of the Lagrangian function with respect to two decision variables, Q_i^{DA} . Optimality conditions of the retailer are:

$$\left(\begin{array}{l} \frac{dL_i}{dQ_i^{DA}} \geq 0 ; Q_i^{DA} \geq 0 \text{ and } \frac{dL_i}{dQ_i^{DA}} \cdot Q_i^{DA} = 0 \\ (\bar{Q}_i^R - Q_i^{DA}) \cdot \lambda_4 = 0 \\ \bar{Q}_i^R \geq Q_i^{DA} \\ \lambda_4 \geq 0 \\ \text{equations (24-27)} \end{array} \right)$$

To solve this non-linear MCP model, we now develop and rearrange the objective function (21) by integrating best-reply quantities for the second and third periods given by equilibrium constraints (24-27), we obtain the following new objective function:

$$E_w(Profit) = G + L \cdot Q_h^{DA} + T \cdot Q_l^{DA} + U \cdot ((Q_h^{DA})^2 + (Q_l^{DA})^2) - V \cdot (Q_h^{DA} \cdot Q_l^{DA}) \quad (28)$$

Where G, V, U, L and T are aggregated parameters and described as follow:

$$G = E_w \left((\gamma \cdot (m - Pen_w) - c) \cdot \bar{Q}^{RT} \right) + E_w \left(-n \cdot Pen_w \cdot \bar{Q}^{RT^2} \right) \quad (29)$$

$$V = -n \left(\frac{\gamma \cdot n}{\bar{\Delta}\beta} \right)^2 \cdot (E_w(Pen_w)) \quad (30)$$

$$U = \frac{1}{2} \left(\frac{\gamma \cdot n}{\bar{\Delta}\beta} \right)^2 - n \quad (31)$$

$$L = m(E_w(Pen_w) - 1) + n \cdot \gamma \cdot \bar{Q}_h^{RT} + n \cdot \gamma \cdot (E_w(Pen_w) \cdot \bar{Q}_h^{RT}) - \frac{\gamma \cdot n}{\bar{\Delta}\beta} \cdot \Delta \bar{Q}_i^{RT} \quad (32)$$

$$T = m(E_w(Pen_w) - 1) + n \cdot \gamma \cdot \bar{Q}_l^{RT} + n \cdot \gamma \cdot (E_w(Pen_w) \cdot \bar{Q}_l^{RT}) + \frac{\gamma \cdot n}{\bar{\Delta}\beta} \cdot \Delta \bar{Q}_i^{RT} \quad (33)$$

Pen_w : Penalty factor, α if real-time negative imbalance and $\frac{1}{\alpha}$ if positive imbalance.

The new objective function is quadratic, twice differentiable and concave since we can verify that the parameter $U < 0$. Indeed, the slope of the marginal cost function is usually very shallow, close to zero and certainly very low compared to $\bar{\Delta}\beta$ which signals the energy consumers' value differential between peak and off-peak hours. So the first term in U is lower than n .

Grouping the objective function (28) and constraints (22-23) leads to a MCP problem with a concave objective function and linear constraints. Its optimal solutions are then obtained:

$$\begin{cases} Q_h^{DA*} = \frac{V \cdot T + 2 \cdot U \cdot L}{V^2 - 4 \cdot U^2} & (34) \\ Q_l^{DA*} = \frac{V \cdot L + 2 \cdot U \cdot T}{V^2 - 4 \cdot U^2} & (35) \end{cases}$$

This equilibrium holds because the non-negativities of the optima are verified since $L \cdot V \cdot \left(\frac{V \cdot L + 2 \cdot U \cdot T}{V^2 - 4 \cdot U^2} \right) \geq 0$ and $T \cdot V \cdot \left(\frac{V \cdot T + 2 \cdot U \cdot L}{V^2 - 4 \cdot U^2} \right) \geq 0$. The second terms of the last conditions are positive since V is negative, whereas L and T are positive. However, if $Q_i^{DA*} > \bar{Q}_i^{RT}$, the optimal solution will be $Q_i^{DA*} = \bar{Q}_i^{RT}$. Therefore, there are interior solutions (34-35) while $Q_i^{DA*} = \bar{Q}_i^{RT}$ in a specific parameters configuration. The sensitivity analysis done in Section 4 gives a more practical understanding of the two equilibria offering an overall view of retailer strategy.

We can now find the solutions to the sub-problems of the game. Optimal retailer's balancing volumes in real-time and real consumer demand served to the retail market are then:

$$Q_{h,w}^{*R} = \bar{Q}_h^{RT} - \frac{\gamma \cdot n}{2 \cdot \bar{\Delta}\beta} \left(\frac{T-L}{V+2U} \right) \quad (36)$$

$$Q_{l,w}^{*R} = \bar{Q}_l^{RT} + \frac{\gamma \cdot n}{2 \cdot \bar{\Delta}\beta} \left(\frac{T-L}{V+2U} \right) \quad (37)$$

Real-time decisions are given by:

$$Q_{rep,w}^{*RT} = \frac{T-L}{V+2U} \quad (38)$$

$$Q_{h,w}^{RT} = \bar{Q}_i^{RT} - \frac{\gamma \cdot n}{2 \cdot \bar{\Delta}\beta} \left(\frac{T-L}{V+2U} \right) - \frac{V \cdot T + 2 \cdot U \cdot L}{V^2 - 4 \cdot U^2} \quad (39)$$

$$Q_{l,w}^{RT} = \bar{Q}_i^{RT} + \frac{\gamma \cdot n}{2 \cdot \bar{\Delta}\beta} \left(\frac{T-L}{V+2U} \right) - \frac{V \cdot L + 2 \cdot U \cdot T}{V^2 - 4 \cdot U^2} \quad (40)$$

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