Optimal control of storage and short-term price formation in electricity markets

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CEEM PhD Conference 04/04/2024

Motivation

Context

- Necessity to **massively replace** fossil-fired power plants by renewable technologies $\longrightarrow$ intermittent: this requires a large-scale use of electricity storage
- Pumped Hydroelectric Energy Storage (PHES) = 86% of total electricity storage in the world

Aim of this work

- Define a realistic and tractable model to study the problem of the optimal strategy for a price taker PHES
- Study its impact on the short-term equilibrium in the electricity market in different frameworks

Literature

Small literature

- Literature on optimal control of storage (Carmona and Ludkovski, Cruise and Zachary, etc.)
- A review on the development of PHES by Barbour et al
- Price formation on electricity market (Fujii and Takahashi or Aïd et al with Mean Field Games), usually implies the study of a Forward-Backward Stochastic Differential Equation

Overview

Plan

Storage

Consider a PHES whose state process, noted Q, represents the **tank level** and follows the SDE:

$$dQ_t = -q_t dt + \rho dW_t^1. \quad (1)$$

q_t, the control process, is the **withdrawal rate** (positive or negative). Here, ρ describes the random amount of energy lost/gained because of external factors (example: drought).
The price P is considered stochastic and exogenous. The stochastic optimal control problem for a single agent reads:

$$V(0, Q_0) = \inf_q \mathbb{E}\left[\int_0^T -(P_s q_s - \alpha q_s^2) + \frac{\beta}{2}(Q_s - Q_0)^2 ds + \frac{\gamma}{2}(Q_T - Q_0)^2\right].$$

with γ,β,α strictly positive numbers.

Storage

Proposition

The **closed loop** optimal control process can be expressed as follows :

$$q_t = \sqrt{\frac{\beta}{\alpha}} f(t,T)(Q_t - Q_0) - \mathbb{E}\left[\int_t^T f_1(t,T,s)\frac{P_s}{2\alpha} ds | \mathcal{F}_t\right] + \frac{P_t}{2\alpha}$$

with $u = \frac{\sqrt{\alpha\beta}-\gamma}{\sqrt{\alpha\beta}+\gamma}$ and f, f_1 auxiliary functions depending on α, β, γ and T.

Overview

Global framework

With this knowledge, the focus shifts to viewing the electricity market as a platform for interaction between the energy demand D and three types of players:

Players

- **Renewable producers**: they always bid their full, stochastic, capacity R_t
- **Conventional producers**: they use a supply function $C(P_t)$, supposedly known and depending only on the electricity price
- **Storage facilities**: they have their optimal strategy found earlier q_t.

The price process is defined in this way :

$$P_t = \inf \{P : D_t \leq R_t + C(P) + q_t\} \wedge \bar{P}$$

with $\bar{P}$ an upper bound for the electricity price. Here there is only one big storage representing the aggregation of every small player.

Overview

Overview

Assumptions

- We ignore the random events that could affect the tank level of the PHES.
- There is no renewable production
- The energy demand is deterministic
- C is a linear function, $C(P) = C_0 + CP$
- the cap price is $\bar{P} = \infty$

$\longrightarrow$ q is deterministic
$\longrightarrow$ P is deterministic

Toy model

The main positive aspect of this toy model is that we can derive an explicit expression for the withdrawal rate and the electricity price :

$$q_t = -c_1 \frac{\beta}{\alpha} \cosh(\sqrt{\frac{\beta}{\alpha}} t) + \frac{P_t}{\alpha} + \int_0^t \sqrt{\frac{\beta}{\alpha}} \sinh(\sqrt{\frac{\beta}{\alpha}}(t-s)) \frac{P_s}{2\alpha} ds$$

$$P(t) = \frac{D_t - C_0 - \frac{\sqrt{\frac{\beta}{\alpha}}}{2\alpha} G(t) + c_1 \frac{\beta}{\alpha} \cosh(\sqrt{\frac{\beta}{\alpha}} t)}{C + \frac{1}{2\alpha}} \wedge \bar{P}$$

with c_1 a constant depending on α, β and γ, G a given function.

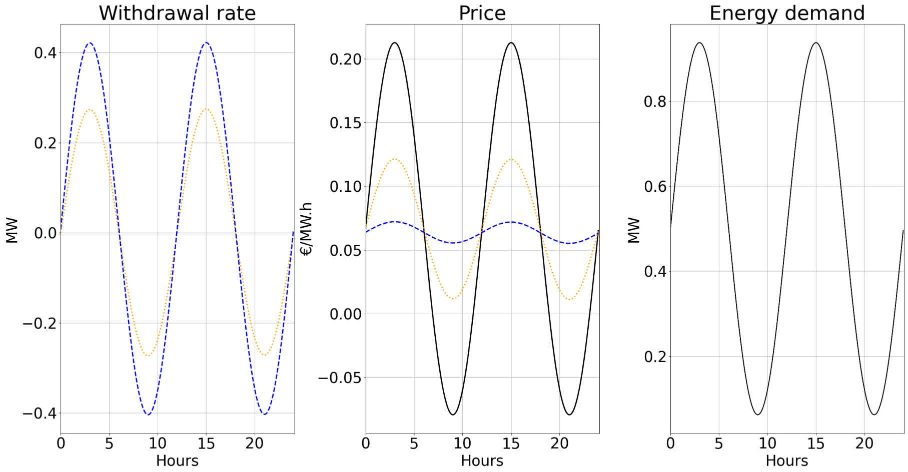
Proposition

For any $\beta, \gamma > 0$,

$$\forall t < T, |\frac{\partial P_t}{\partial t}| \underset{\alpha \longrightarrow 0}{\longrightarrow} 2\beta \int_0^t (D_s - C_0) \sinh(\sqrt{2C\beta}(t-s)) ds \qquad (2)$$

Numerical illustration

Blue : Abundant storage, Orange : Medium amount of storage, Black : no storage.



Overview

Let us assume the presence of N different categories of storage, each with unique parameters and characteristics.

Agent j

$$dQ_t^j = -q_t^j dt + \rho^j dW_t^{1,j}$$

He solves the following problem:

$$V(0, Q_0^j) = \inf_{q_t^j} \mathbb{E} \int_t^T -(P_t q_t^j - \alpha^j (q_t^j)^2) + \frac{\beta^j}{2}(Q_t^j - Q_0)^2 dt + \frac{\gamma^j}{2}(Q_T^j - Q_0^j)^2$$

and his optimal strategy is

$$q_t^j = \sqrt{\frac{\beta^j}{\alpha^j}} f^j(t, T, t)(Q_t^j - Q_0^j) - E\left[\int_t^T f^j(t, T; r)\frac{P_s}{2\alpha^j} ds | \mathcal{F}_t^j\right] + \frac{P_t}{2\alpha^j}$$

As before, we look at the electricity market as a platform for interactions between three types of players.

Players

- **Renewable producers**: they always bid their full, stochastic, capacity R_t
- **Conventional producers**: they use a supply function $C(P_t)$, supposedly known and depending only on the electricity price
- **Storage facilities**: they have their optimal strategy found earlier q_t.

In this case, the price process is defined in this way:

$$P_t = \inf \left\{ P : D_t \leq R_t + C(P) + \sum_j q_t^j \right\} \wedge \bar{P}$$

Theorem

Suppose that the residual demand $\tilde{D}_t := D_t - R_t$ can be written in this way :

$$d\tilde{D}_t = \mu(t, \tilde{D}_t)dt + \sigma(t, \tilde{D}_t)dW_t^2, \quad \tilde{D}_0 = \bar{D}_0$$

where μ and σ satisfy Lipschitz-type and non-degeneracy conditions. There exists a unique price process resulting from the above definition in $[0, T]$.

FBSDE

Forward Backward Stochastic Differential Equation

The processes X, Y and Z are solution to the following **Forward-Backward Stochastic Differential Equation** (FBSDE):

$$\begin{cases} \forall t \in [0,T], X_t = x + \int_0^t b(s, X_s, Y_s)ds + \int_0^t \sigma(s, X_s)dW_s \longrightarrow \textit{Forward} \\ \forall t \in [0,T], Y_t = Y_T + \int_t^T f(s, X_s, Y_s)ds - \int_t^T Z_s dW_s \longrightarrow \textit{Backward} \end{cases} \quad (3)$$

with b, σ and f deterministic functions.

Sketch of the proof

For all $j \leq n$, we can rewrite q^j as follows:

$$q_t^j = \sqrt{\frac{\beta^j}{\alpha^j}} f^j(t,T,t)(Q_t^j - Q_0^j) + \frac{P_t}{2\alpha^j} - F_1^j(t,T)Y_t^{j,1} + F_2^j(t,T)Y_t^{j,2}$$

with Y_1 and Y_2 satisfying BSDEs and some functions F_1^j and F_2^j depending solely on α, β, γ.

We can prove that looking for the equilibrium situation is equivalent to studying the following FBSDE :

$$\begin{cases} P_t = \inf\left\{P : \tilde{D}_t \leq C(P_t) + \sum_{j=1}^n q_t^j\right\} \wedge \bar{P} \\ d\tilde{D}_t = \mu(t,\tilde{D}_t)dt + \sigma_t(t,\tilde{D}_t)dW_t^2 \\ \forall j \leq n \\ dQ_t^j = -q_t^j dt + \rho^j dW_t^{(1,j)} \\ dY_t^{j,1} = -e^{-t\sqrt{\frac{\beta^j}{\alpha^j}}} \dfrac{P_t}{2\alpha^j} dt + Z_t^{j,1} dW_t, \\ dY_t^{j,2} = -e^{t\sqrt{\frac{\beta^j}{\alpha^j}}} \dfrac{P_t}{2\alpha^j} dt + Z_t^{j,2} dW_t, \quad Y_T^{j,1} = Y_T^{j,2} = 0 \end{cases}$$

Numerical illustrations

Residual demand is calibrated using an Ornstein Uhlenbeck. Indeed, define $(A_t)_t$ such that

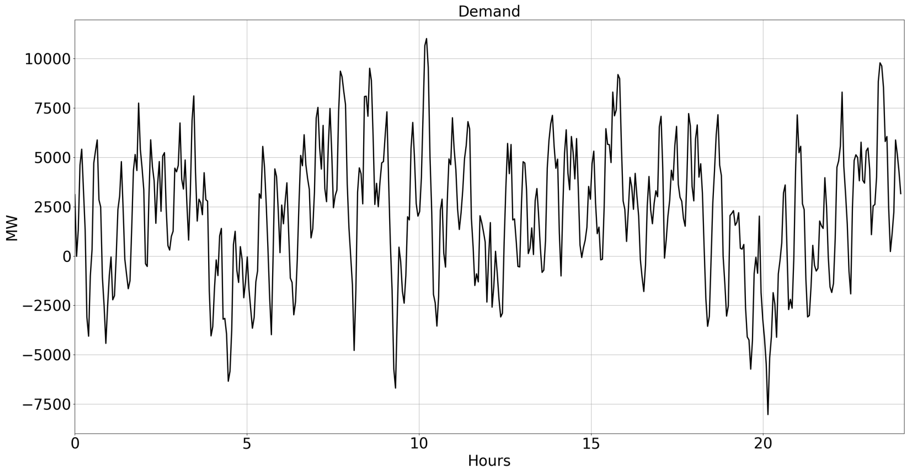
$$A_t := \tilde{D}_t - \bar{D}_t$$

with $\bar{D}$ being the hourly mean of the residual demand. A is then taken to be an Orstein-Uhlenbeck process:

$$A_t = A_0 - \int_0^t \theta(A_t - \mu)dt + \int_0^t \sigma W_t$$

$\Longrightarrow$ Calibrated using French data over 30 days from ENTSOE.
The **conventional supply function** is taken to be linear $C_0 + CP$.
$\Longrightarrow$ C_0 is an adjustment variable.

Numerical Illustration



Introduce the following FBSDE :

$$\begin{cases} \forall t \in [0, T], X_t = x + \int_0^t b(s, X_s, Y_s) ds + \int_0^t \sigma(s, X_s) dW_s \\ \forall t \in [0, T], Y_t = \int_t^T f(s, X_s, Y_s) ds - \int_t^T Z_s dW_s \end{cases} \tag{4}$$

with b, σ and f deterministic functions.
There exists u and v such that (X, Y, Z) are connected through the following formulas :

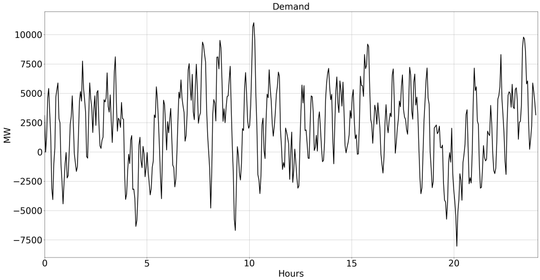
$$Y_t = u(t, X_t), \quad Z_t = v(t, X_t)$$

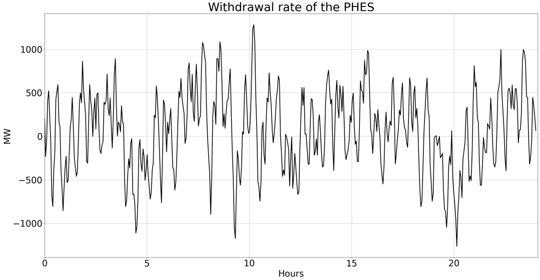
The numerical scheme for a time interval divided into p segments can be expressed as follows, with $u_i^{p,0} = 0$, at the m^{th} iteration:

$$\begin{cases} X_0^{p,m} = x \\ X_{i+1}^{p,m} = X_i^{p,m} + b(t_i, X_i^{p,m}, u_i^{p,m-1}(X_i^{p,m}))h + \sigma(t_i, X_i^{p,m})\Delta W_{i+1}, \\ Y_p^{p,m} = 0, \\ Z_i^{p,m} = \frac{1}{h}\mathbb{E}[Y_{i+1}^{p,m}\Delta W_{i+1}|\mathcal{F}_{t_i}] \\ Y_i^{p,m} = \mathbb{E}[Y_{i+1}^{p,m} + f(t_i, X_i^{p,m}, Y_{i+1}^{p,m})h|\mathcal{F}_{t_i}] \\ u_i^{p,m}(X_i^{p,m}) = Y_i^{p,m} \end{cases}$$

with $h = \frac{T}{p}$.

Numerical illustrations





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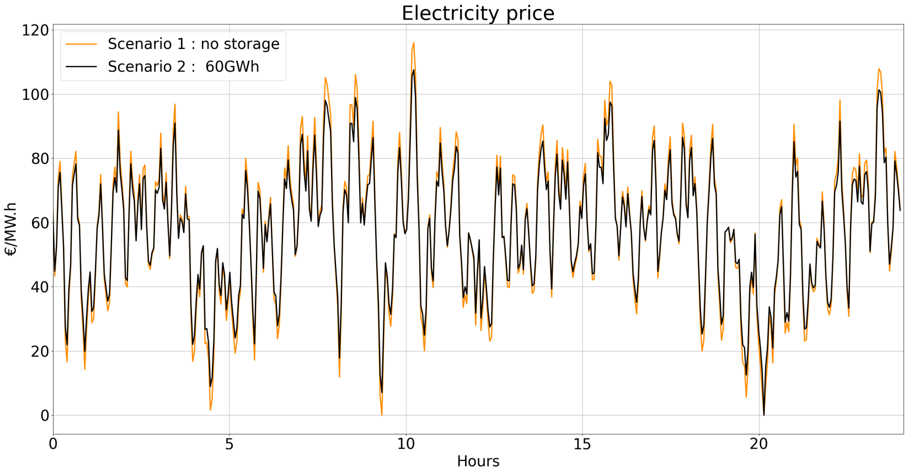

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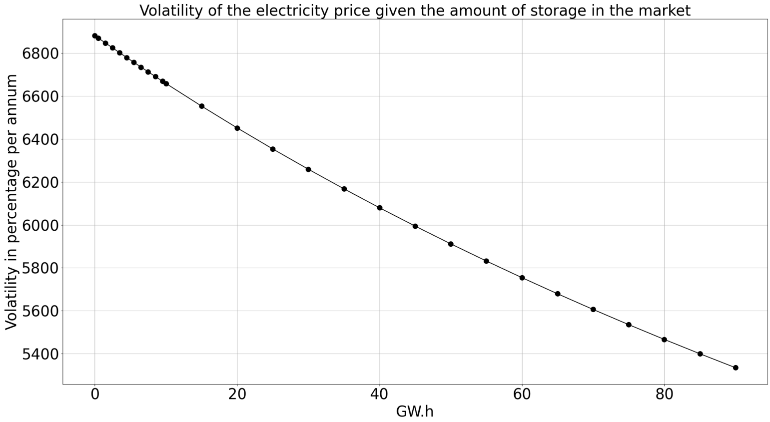
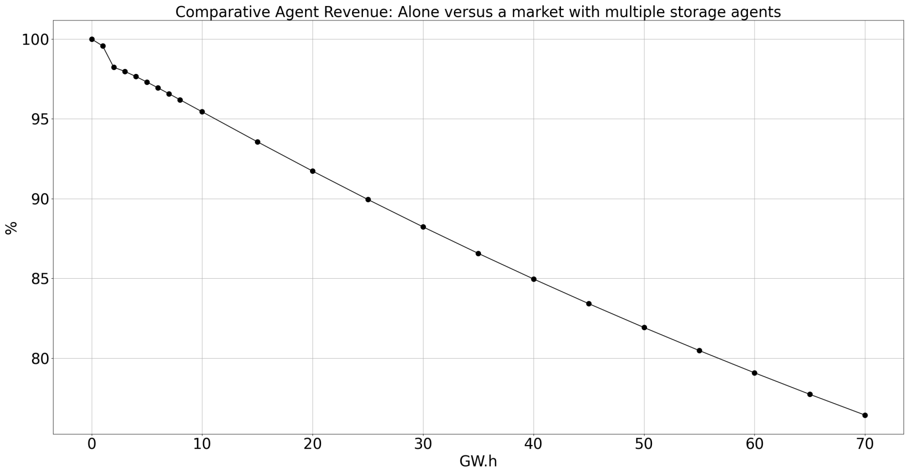


Numerical Illustration



Wind Farm

Is it worth it for a wind farm owner to invest in storage?

$\Longrightarrow$ Consider a windfarm project with a capacity of 100MW.

Revenues without storage: 44152€.

Slightly modify our model to integrate the possibility to have a storage combined with an external source of energy, such as a wind farm:

$$dQ_t = (-q_t + \kappa_t)dt + \rho_t dW_t^{1,1}$$

Revenues with a storage of 100MW.h: 52367€.

Conclusion

Conclusion

- We introduced a tractable model to derive with classic methods an explicit expression for the optimal strategy of a storage system, considering both deterministic and stochastic exogenous price processes.
- We proved the existence and uniqueness of the price process resulting from short-term equilibrium between the energy demand, renewable production, conventional producers, and storage players.
- We observed in the deterministic case that increasing storage capacity led to a compression of electricity prices
- The more storage there is on the electricity market, the less volatility there is from renewable energy producers.

Conclusion

On going work

- Generalize the Brownian noise of the storage model.
- Introducing a mean-field framework to better describe the variety of storage agents.

Thank you!