# The impact of auction quantity on the outcomes in a multi-unit procurement auction for RES

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# Outlines

# Context

Procurement auction for RE supply on the rise:

- countries using auction rise from 6 in 2005 to at least 67 by mid 2016 (IRENA 2015, 2017).
- worldwide RE capacity of 111 GW was auctioned in 2017-2018 $\approx$ 222 billion USD (Ehrhart et al., 2020).
- widely used in both developing and developed countries.

$\Longrightarrow$ Procurement auction's popularity gradually dominating feed in tariff (FIT) scheme.

# Why auction popular? I

Procurement auction could solve the challenge FIT was facing.

- FIT sets price manually.

Whereas,

- Bid competitions from auction allows for market price discovery.

$\Longrightarrow$ Very important as RE technology is constantly improving implying a decreasing cost of RE generation. A market price discovery mechanism is essential!

# Why auction popular? II

- Standard auction ensures RE development contracts are allocated to the most efficient developers.

$\Longrightarrow$ Efficiency of contract allocations.

- Auctioneers have various choices to set market mechanisms to procure RE plants.

# Auction is also not perfect

- Potential bidders (RE developers) have to pass costly pre-qualification requirements.
- Auction participants pay this cost even if they do not win any contracts in the end.

$\Longrightarrow$ Barrier to participation. Only developers with expected payoff from the auction offsetting this cost would join.
$\Longrightarrow$ Risk of not enough competition to reveal the competitive market price.

Meanwhile, market price discovery is the heart of auction!!

# What can we do?

- Costly pre-qualification requirements are a necessity to minimize risk of project delays.
- We have to take high bid preparation costs and low auction participation as given.

$\Longrightarrow$ Look for a tool we can make use of.

$\Longrightarrow$ We found auction quantity to be quite a useful tools.

**Intuition:**

- More auction quantities (RE contracts), higher probability of being among the winners, higher expected payoff of joining. More incentive to participate.

- More goods to be won also mean less pressure to bid competitively.

$\Longrightarrow$ The trade off points to the possibility of an optimal point of auction quantity.

# Research Question

What is the optimal auction quantity auctioneers should set with regards to their objective?

We currently focuses on two popular objectives among RE auctioneers.

1. Auctioneer's expected surplus.
2. Social welfare

# Literature

- The earlier literature of procurement auction with participation costs started **Samuelson (1985)**. He derived for optimal reserve price for maximizing social welfare and auctioneer surplus in a single unit setting.
- **Stegemann (1996)** found that second price auction with reserve price set to the level of auctioneer's own valuation was ex-ante optimal.
- **Menezes and Monteiro (2000)** derived an optimal revenue maximizing sales mechanism whose translation into the procurement setting suggested to refund the bid preparation costs to all participating bidders or to conduct a second price auction whose reserve price is set to the highest cost participants.
- **Ehrhart et al. (2020)** analyzed the recommendation of using endogenous rationing to solve low participation problem and proved that it was indeed suboptimal with regards to prevalent objectives. This papers also found optimal designs for four prevalent objectives and compare them in one place.

Literature dealt with costly bid preparation, but mostly focused on the mechanism design aspect of the problem. Some that differed tend to analyze the optimal reserve price. Meanwhile, we will focus on optimal auction quantity.

# Model setup I

- $k$ units of good offered to be procured, where $k \geq 1$. $n$ risk neutral potential bidders each with a single unit supply, $n \geq 1$.
- Denote $m$ the number of participating firm, so $m \leq n$. We only consider pure strategies in our model.
- Each firm $i$'s private costs $x_i$ is identically and independently distributed from the distribution $F$ with density $f$ fully supported on $[\underline{x}, \bar{x}]$, where $0 \leq \underline{x} \leq \bar{x}$.
- The bid preparation cost incurred by participating firms is $c > 0$. A firm knows $c$ and their private costs $x_i$ when they decide about their participation.

# Model setup II

We follow the multi-unit STD format, thus the following properties are assumed:

1. When bidding, the firms know $n$ but do not know $m$.
2. Bids may not exceed a reserve price $r \in \mathbb{R}+, r > \underline{x} + c$,set by the auctioneer.
3. The $k$ lowest bids win if $m \geq k$, all other firms obtain nothing. If $m < k$, all $m$ bids win.
4. The reserve price $r$ is the maximum payment from the auction, and for each firm there exists a bid such that the firm's payment with this bid is $r$ if $m \leq k$.
5. Participating firms apply a symmetric bidding function that is non-decreasing in $x_i$ .

In this setup, we will assume uniform price auction without loss of generality due to the revenue equivalence property of STDs.

# Expected payoff and participation

- Let $F(k, n)$ denotes the distribution function of the $k$-th lowest of $n$ independent signals and let $X(k, n)$ denote the associated random variable. Thus,

$$F_{(k,n-1)} = \sum_{i=k}^{n-1} \binom{n-1}{i} F(x)^i (1 - F(x)^{n-1-i}. \tag{1}$$

- Denote the cutoff firm $\hat{i}$ with realized cost $\hat{x} < \bar{x}$. $\hat{x}$ can be uniquely determined by the function:

$$\Pi(\hat{x}, r, c, n) = (r - \hat{x})(1 - F_{(k,n-1)}(\hat{x})) - c \tag{2}$$

$$= (r - \hat{x}) \sum_{i=0}^{\min(k,n)-1} \binom{n-1}{i} F(\hat{x})^i (1 - F(\hat{x})^{n-1-i} - c = 0 \tag{3}$$

- For each $k$ value, there exist a unique $\hat{x}$ value associated to this $k$ auction quantity.

# The objective functions

Define the value of the renewable energy support as $v \geq r$.

- The auctioneer expected surplus:

$$\Pi_0 = (v-r)\sum_{i=1}^{\min(k,n)} i\binom{n}{i}F(\hat{x})^i(1-F(\hat{x})^{n-i} + k(v-\mathbb{E}[X_{(k+1,n)}|X_{(k+1,n)} \leq \hat{x}])F_{(k+1,n)}(\hat{x}). \quad (4)$$

- The expected social welfare:

$$W = -ncF(\hat{x}) - \sum_{i=1}^{\min(k,n)} \mathbb{E}[X_{(i,n)}|X_{(i,n)} \leq \hat{x}]F_{(i,n)}(\hat{x}) + v(\min(k,n) - \sum_{i=1}^{\min(k,n)-1} (\min(k,n)-i)\binom{n}{i}F(\hat{x})^i(1-F(\hat{x})^{n-1}) \quad (5)$$

# Attempts at algebraic analysis

Due to the discrete nature of $k$, we cannot take the derivative of of the objective function with respect to $k$.

- An alternative approach would be to directly study the sign of $W(k) - W(k-1)$ at each $k$. However, no algebraic simplification available.
$\Longrightarrow$ Approach this problem numerically.

# Numerical methods

For auctioneer expected surplus:

$$\Pi_0 = (v - r) \sum_{i=1}^{\min(k,n)} i \binom{n}{i} F(\hat{x})^i (1 - F(\hat{x})^{n-i} + kvF_{(k+1,n)}(\hat{x}) - k(k+1) \binom{n}{k+1} \int_{x}^{\hat{x}} xf(x)F(x)^k (1 - F(x))^{n-k-1}\, dx \tag{6}$$

For expected social welfare:

$$W = -ncF(\hat{x}) - \sum_{i=1}^{\min(k,n)} i \binom{n}{i} \int_{x}^{\hat{x}} xf(x)F(x)^{i-1} (1 - F(x))^{n-i}\, dx + v(\min(k,n)) - v \sum_{i=1}^{\min(k,n)-1} (\min(k,n) - i) \binom{n}{i} F(\hat{x})^i (1 - F(\hat{x})^{n-1}) \tag{7}$$

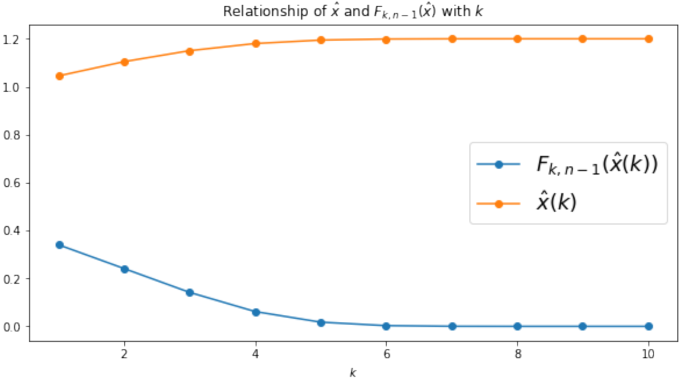
# Parametrization

We let $F(x)$ be uniformly distributed on $[1, 2]$ meaning $\underline{x} = 1, \bar{x} = 2$.
Thus,

$$F(\hat{x}) = \hat{x} - 1, 1 - F(\hat{x}) = 2 - \hat{x}, f(\hat{x}) = 1$$

We initially let, $c = 0.3, r = 1.5, v$ takes value in $[1.5, 1.8], n = 10, k$ takes value in $[1, n]$.

# Preliminary Test




- Relationships of how $k$ affects $\hat{x}$ and $F_{(k,n-1)}(\hat{x})$.
- Expect $k$ to have a non-negligible effect on the objective functions.

# Some prior expectations

- Increasing $k$ increases $\hat{x}$ . This implies an increase in the expected number of goods acquired by the auctioneer which would contribute positively to the objective function.
- However, an increasing $\hat{x}$ also means higher cost firms would be among the participants who bid in the auction.

# Numerical results I

**Case** $v > r$

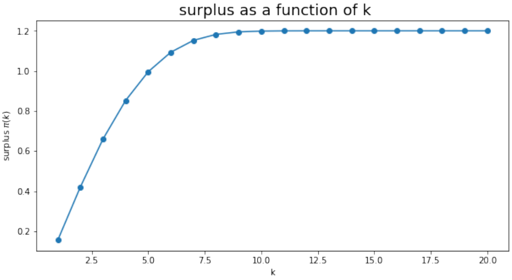



Figure 1: $\Pi_0$ as a function of $k$

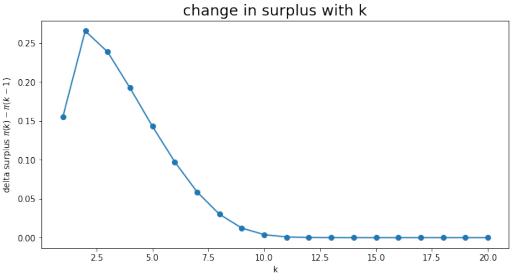
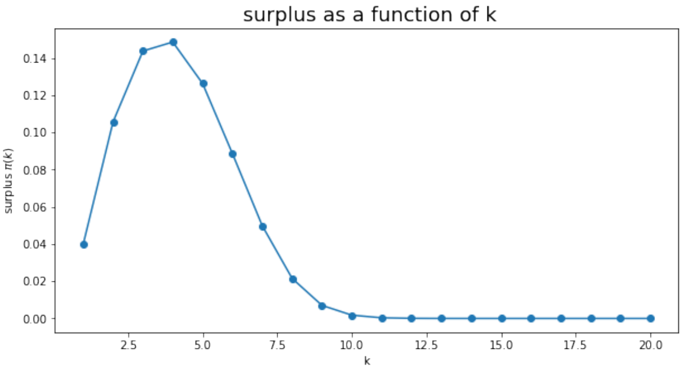



Figure 2: change in $\Pi_0$ as a function of $k$

# Numerical results II

**Case $v = r$**



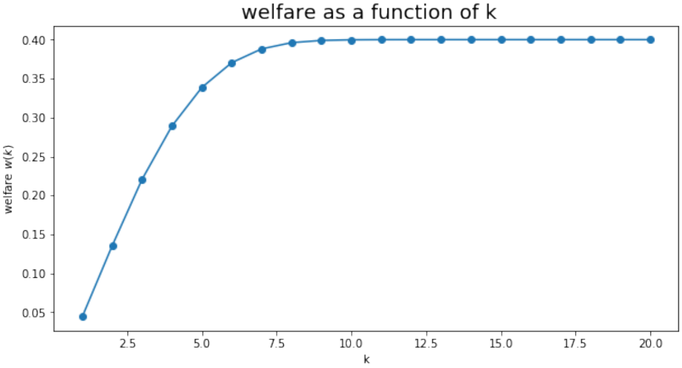

- Concave $\Pi_0$. Max at around $k = 4$ when $n = 20$.

# Some prior expectations

Social welfare makes up of three main components: the expected total costs of participation, the expected total costs of producing the goods and the expected total value of obtaining the goods.

- Increasing $k$ increases the participation probability $F(\hat{x})$, so this should increase the expected total participation costs which negatively affect the welfare function.
- As $\hat{x}$ increases with $k$ meaning more costly firms would participate, we expect $E[X_{(i,n)}|X_{(i,n)} \leq \hat{x}]$ to increase with $k$. However, it is multiplied by $F_{(k,n)}(\hat{x})$ which decreases with $k$. Ambiguous effects.
- The final terms, the expected quantity acquired, increases with $k$ if $m \leq k$. The opposite effect happens if $m > k$.

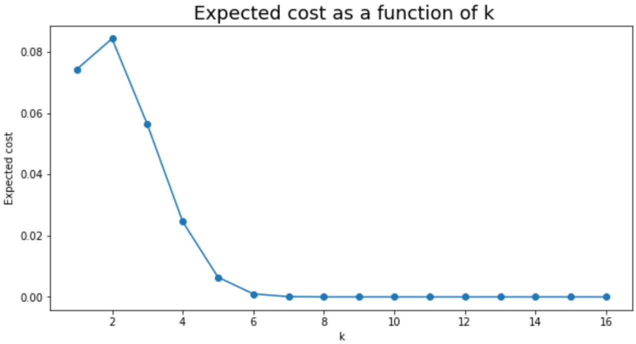
# Numerical results




- Quite surprising as we would expect the welfare function to start decreasing with $k$ after some $k$ sufficiently big.

# Understanding why

- Graph $E[X_{(i,n)}|X_{(i,n)} \leq \hat{x}]F_{(i,n)}(\hat{x})$ in terms of $k$.




- It is an decreasing with $k$!

# Conclusions

For auctioneer's expected surplus:

- When $v > r$, expected surplus increases with $k$.
- When $v = r$, expected surplus is concave and so there exists an optimal auction quantity $k^*$ .

For expected social welfare:
Social welfare function increases with $k$. Need further works to understand exactly why.

# Limitation

- The choice of $F$ to be uniformly distributed which implies every $x_i$ has equal chance of happening.
- Numerical result shows only local extreme points. Can't verify the patterns observed here hold outside the tested values.
Might be why some results are counter-intuitive.
Possible to get a different insight with a different parametrization.

# Extension

- Choose the parameter values that conform to the optimal mechanism that have been found in previous literature. With this optimal setup, we want to understand the effects of $k$.

- Work out the result theoretically.

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