A mean-field game model of electricity market dynamics

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Introduction

- With decarbonization, increasing share of renewable generation and closure of coal-fired power plants, but system more relying on gas power plants
- Recent crisis showed the interest of modelling uncertainty and a need for a market design reform
- But many difficulties to fully represent the current market dynamics
- Perfect competition (equivalent to social planner) is still the main paradigm used, neglecting market bias

Research question

How to represent entry and exit dynamics on the electricity market taking into account possible strategic behaviour?

Main contributions

- Build a long-term model for the dynamics of the electricity industry to describe energy transition
- Take into account the role of gas a medium-term substitute for coal, with endogenous gas price
- Introduction of strategic interactions with cost uncertainty and agents heterogeneity: agents anticipate other agents' actions

Related Literature

- Electricity market models classified into 3 categories (Ventosa et al. 2005):
  (1) Market equilibrium models: tractable equilibrium concept with a reduced form
    - A lot of simplifying hypothesis (number of players, strong homogeneity of agents...) to get an equilibrium
    - Often static and deterministic models

  (2) Simulation and (3) Optimization models: engineering models allowing to represent large power systems, with strong optimization tools
    - No clear representation of strategic interactions
    - Complex to compute, analyze and interpret

- Mean-Field Game is an dynamic equilibrium model with many player AND with a tractable solution
- Relaxed assumptions allowing to take into account uncertainty, heterogeneity of players, endogenous fuel prices...

A Mean-Field Game Model for Entry/Exit on the electricity market

The agents

- Each electricity producer j uses a technology of type i, from two categories:
- Conventional power plants:
  - One unit of capacity, bid a fraction ξ of this capacity
  - Random costs component (CIR)

$$C_t^{ij}(\xi) = \underbrace{f_i e_{k(i)} P_t^C}_{\text{Carbon cost}} + \underbrace{f_i P_t^{k(i)}}_{\text{Fuel cost}} + \underbrace{Z_t^{ij}}_{\text{Random cost}} + \underbrace{c^i(\xi)}_{\text{Operating cost}} \quad (1)$$

- Renewable power plants:
  - Random capacity factor
  - Bid the entire possible production on the market

▸ Agents random component

Price formation

Electricity Price:

- Agents offer electricity quantities on the market
- Market matches exogenous demand with supply from renewables and conventional producers
- Conventional producers select fraction ξ for revenue maximization
- Insufficient supply vs. demand leads to market failure; electricity price caps at P^*

Fuel Price:

- Exogenous supply function for each fuel $k(i)$.
- Fuel price results from matching supply function with fuel consumption for electricity production.

▸ Price equations

Entry on the Market

- Potential producers aim for optimal market entry τ_1 and exit τ_2 times to:
  - Maximize expected revenues conditional on entry and exit times
- Conventional producers already in the market evaluate optimal exit time τ_2 to:
  - Maximize expected revenues conditional on exit time
- Includes construction time, lifetime $\lambda(i)$, investment cost $K_{t,i}$ for technology i, scrap value $\tilde{K}_i$.
- Accounts for fixed cost $\kappa_{t,i}$, capital costs decay rate γ_i.

▸ Maximization Programs

Nash Equilibrium

- Classical Nash Equilibrium: Agent j chooses strategies (τ_1^j, τ_2^j) without incentive to deviate, considering others' strategies
- **Challenging to compute for numerous players!**

Nash Equilibrium

- Classical Nash Equilibrium: Agent j chooses strategies (τ_1^j, τ_2^j) without incentive to deviate, considering others' strategies
- **Challenging to compute for numerous players!**
- Mean-Field Theory: Replace class of agent j by an infinite population of agents of type j, described by a distribution $m_t^j(da, dx)$ of ages and costs
- Mean-Field Nash Equilibrium: A representative agent of class j has no incentive to deviate given distributions $m_t^{-j}(da, dx)$

Mean-Field formulation

Linear programming approach

- Each agent maximises its expected gains as a linear function of the occupation measure of the population in the market, allowing to use the linear programming approach
- Addition of linear constraints on the measures to respect the stochastic process dynamics for cost functions and renewable capacity factor

- MF Nash equilibrium a sequence of entry/exit measures and price functions such that:
  1. For each $i = 1, \ldots, N$, measures maximize the conventional producers program
  2. For each $i = N + 1, \ldots, \overline{N}$, measures maximize the renewable producers program
  3. For each t, the price vector is the solution of the system of demand matching supply

Formal definition of Nash equilibrium | Numerical resolution

Numerical illustration

Calibration

- Calibration basis: German data over a 25-year horizon from 2018.
- Technologies covered:
  - Coal: Exit strategy.
  - Gas: Strategies for both exit and entry.
  - Wind: Entry strategy.
- Construction timelines:
  - Renewable projects: 2 years.
  - Gas plants: 4 years.
- Assumption of infinite technology lifetimes.
- Carbon tax trajectory: Increase from 30 to 200.

Example of output (I)

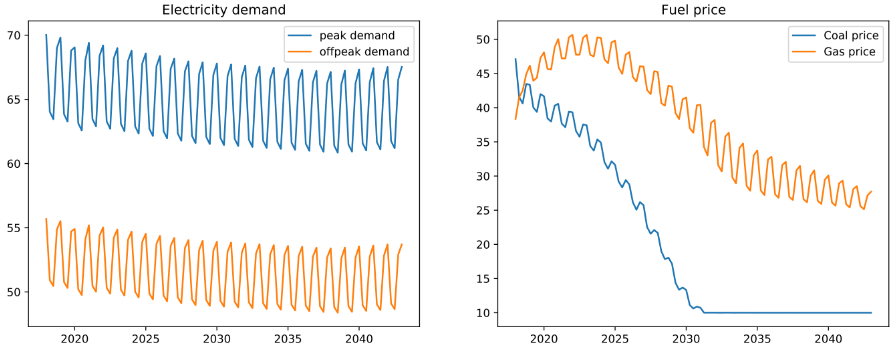

Figure 1: Electricity demand and fuel prices

Example of output (II)

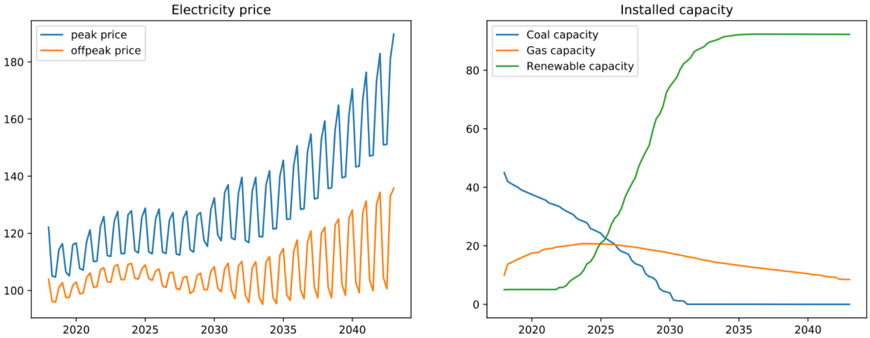

Figure 2: Electricity prices and installed capacities

Example of output (III)

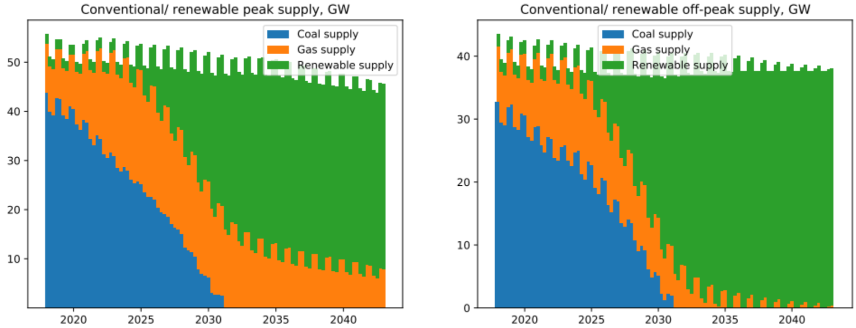

Figure 3: Electricity production mix

Conclusion

- Traditional equilibrium models face computational challenges with large numbers of agents, heterogeneity, and uncertainty
- Mean-Field Games (MFG) provide a robust framework for modelling the electricity market, offering:
  1. Scalability to accommodate numerous participants.
  2. Flexibility to incorporate heterogeneity, uncertainty, and constraints.
  3. Integration of fuel price endogeneity.
  4. More tractable and general solutions compared to simulation approaches.
- The model effectively demonstrates gas's role as an intermediate substitute for coal, until renewable technologies mature

Thank you for your attention

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References I

Ventosa, Mariano et al. (2005). “Electricity market modeling trends”. In: *Energy policy* 33.7, pp. 897–913.

Pricing equations

$$\left(D_t^p - R_t\right)^+ = F_0\left(P_t^p\right) + \sum_{k=1} F_t^k\left(P_t^p, P_t^k\right), \tag{2}$$

$$\text{or } \left(D_t^p - R_t\right)^+ > F_0\left(P_t^p\right) + \sum_{t}^{K} F_t^k\left(P_t^p, P_t^k\right) \quad \text{and} \quad P_t^p = P^* \tag{3}$$

Fuel and price equations

- Fuel price solving:

$$c_p \Psi_t^k \left(P_t^p, P_t^k\right) + c_{op} \Psi_t^k \left(P_t^{op}, P_t^k\right) = \Phi_k \left(P_t^k\right) \tag{4}$$

- Fuel consumption:

$$\Psi_t^k \left(P^E, P^k\right) = \sum_{i:k(i)=k} \sum_{j=1}^{N_i} \lambda_i \left(t - \tau_1^{ij}\right) \mathbf{1}_{\tau_2^{ij} > t} f_i Q_{ij} F_i \left(P^E - f_i e_{k(i)} P^C - f_i P^k - Z_t^{ij}\right), \tag{5}$$

▸ Price formation

Agents production processes

- Renewable capacity factor for agent j with technology i:

$$dS_t^{ij} = \overline{k}^i(\overline{\theta}^i - S_t^{ij})dt + \overline{\delta}^i\sqrt{S_t^{ij}(1 - S_t^{ij})}dW_t^{ij}, \quad S_0^{ij} = \overline{s}_{ij} \tag{6}$$

- Random cost component for agent j with technology i:

$$dZ_t^{ij} = k^i(\theta^i - Z_t^{ij})dt + \delta^i\sqrt{Z_t^{ij}}dW_t^{ij}, \quad Z_0^{ij} = z_{ij} \tag{7}$$

▸ The agents

Agents maximization programs (I)

- Conventional producers instantaneous gain function:

$$\int_0^{\xi^*} \left(p - C_t^{ij}(\xi)\right) d\xi = G_i \left(p - e_{k(i)} P_t^C + P_t^{k(i)} + Z_t^{ij}\right) \tag{8}$$

- Conventional cost function for agent j with technology i:

$$\mathbb{E}\left[\underbrace{\int_{\tau_1}^{\tau_2} e^{-\rho t} \lambda_i(t-\tau_1)\left(G_i\left(P_t - f_i e_{k(i)} P_t^C - f_i P_t^{k(i)} - Z_t^{ij}\right) - \kappa_i\right) dt}_{\text{Market gains}} \right.$$
$$\left. - \underbrace{K_i e^{-(\rho+\gamma_i)\tau_1}}_{\text{Entry cost}} + \underbrace{\widetilde{K}_i e^{-(\rho+\gamma_i)\tau_2}}_{\text{Exit scrap. value}}\right]$$

Game of entry and exit

Agents maximization programs (II)

- Renewable supply function for agent j with technology i:

$$\mathbb{E}\left[\int_{\tau_1}^{\tau_2} \underbrace{e^{-\rho t}\lambda_i\left(t-\tau_1\right)\left(P_t S_t^i-\kappa_i\right)dt}_{\text{Market gains}} - \underbrace{K_i e^{-(\rho+\gamma_i)\tau_1}}_{\text{Entry cost}} + \underbrace{\widetilde{K}_i e^{-(\rho+\gamma_i)\tau_2}}_{\text{Exit scrap. value}}\right] \tag{9}$$

Game setting

Infinitesimal Generator: Conventional

- Conventional cost function process:

$$dZ_t^{ij} = k^i(\theta^i - Z_t^{ij})dt + \delta^i \sqrt{Z_t^{ij}} dW_t^{ij}, \quad Z_0^{ij} = z_{ij} \tag{10}$$

- Associated Infinitesimal generator for a C^2 u function:

$$\mathcal{L}_{ij} u = k^i(\theta^i - z)\frac{\partial u}{\partial z} + \frac{1}{2}(\delta^i)^2 z \frac{\partial^2 f}{\partial z^2}$$

▸ The agents

Infinitesimal Generator: Renewable

- Conventional cost function process:

$$dZ_t^{ij} = k^i(\theta^i - Z_t^{ij})dt + \delta^i \sqrt{Z_t^{ij}} dW_t^{ij}, \quad Z_0^{ij} = z_{ij} \tag{11}$$

- Associated Infinitesimal generator for a C^2 u function:

$$\mathcal{L}_{ij}u = k^i(\theta^i - z)\frac{\partial u}{\partial s} + \frac{1}{2}(\delta^i)s(1-s)\frac{\partial^2 u}{\partial s^2}$$

Constraints

Fuel and price equations

- Fuel price solving:

$$c_p \Psi_t^k \left(P_t^p, P_t^k\right) + c_{op} \Psi_t^k \left(P_t^{op}, P_t^k\right) = \Phi_k \left(P_t^k\right) \tag{12}$$

- Fuel consumption:

$$\Psi_t^k \left(P^E, P^k\right) = \sum_{i:k(i)=k} \sum_{j=1}^{N_i} \lambda_i \left(t - \tau_1^{ij}\right) \mathbf{1}_{\tau_2^{ij} > t} f_i Q_{ij} F_i \left(P^E - f_i e_{k(i)} P^C - f_i P^k - Z_t^{ij}\right), \tag{13}$$

▸ Agents

Introduction of measures

- 2 classes of population for agent of type i
  - Class $\hat{C}_i$: plants which the decision to build has not been taken yet
  - Class C_i: plants under construction or operational
- **Occupation Measure** ($m_i(t)$)
  - **Purpose**: Represents the distribution of active agents over their state space at any given time
- **Entry Measure** (ν_i)
  - **Purpose**: Captures the rate and conditions of new market entrants over time
- **Exit Measure** (μ_i)
  - **Purpose**: Quantifies the rate at which agents withdraw from the market

Mean-Field formulation I

$$m_i^t(da, dx) = \int_{A\times O_i} \nu_0^i(da', dx')\mathbb{E}\left[\delta(a' + t, Z_t^i)(da, dx)\right] \tag{14}$$

$$\mu_i(dt, da, dx) = \int_{A\times O_i} \nu_0^i(da', dx')\mathbb{E}\left[\delta(\tau_2^i, \tau_2^i + a', Z_{\tau_2}^i)(dt, da, dx)\right] \tag{15}$$

$$\nu_i(dt, da, dx) = \nu_0^i(da, dx)\delta_0(dt) + \hat{\mu}_i(dt, dx)\delta_0(da) \tag{16}$$

Mean-Field formulation II

$$\hat{\mu}_i(dt, dx) = \int_{O_i} \hat{\nu}_0^i(dx')\mathbb{E}\left[\delta(\tau_1^i, Z_{\tau_1}^i)(dt, dx)\right] \quad (17)$$

$$\hat{m}_i^t(dx) = \int_{O_i} \hat{\nu}_0^i(dx')\mathbb{E}\left[\delta(Z_t^i)(dx)\right] \quad (18)$$

$$\hat{\nu}_i(dt, dx) = \hat{\nu}_0^i(dx)\delta_0(dt) \quad (19)$$

▸ Linear programming

MFG equations: price equations system

- Conventional supply function:

$$F_t^k\left(P^E, P^k\right) = \sum_{i:k(i)=k} \int_{\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx)\lambda_i(a)F_i\left(P^E - f_i e_k P^C - f_i P^k - x\right) \tag{20}$$

- Fuel consumption is therefore:

$$\Psi_t^k\left(P^E, P^k\right) = \sum_{i:k(i)=k} \int_{\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx)\lambda_i(a)f_i F_i\left(P^E - f_i e_k P^C - f_i P^k - x\right) \tag{21}$$

- Renewable supply function:

$$R_t = \sum_{i:k(i)=k}^{N+\overline{N}} \int_{\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da, dx)\lambda_i(a) \tag{22}$$

MFG equations: optimization functionals

- Conventional gain function:

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da,dx)e^{-\rho t}\lambda_i(a)\left(c_p G_i\left(P_t^p - f_i e_{k(i)} P_t^C - f_i P_t^{k(i)} - x\right)\right.$$
$$\left. + c_{op} G_i\left(P_t^{op} - f_i e_{k(i)} P_t^C - f_i P_t^{k(i)} - x\right) - \kappa_i\right) dt$$
$$-K_i \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \hat{\mu}^i(dt,da,dx)e^{-(\rho+\gamma_i)t} + \widetilde{K}_i \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \mu^i(dt,da,dx)e^{-(\rho+\gamma_i)t}$$

MFG equations: optimization functionals

- Renewable gain function:

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} m_t^i(da,dx)e^{-\rho t}\lambda_i(a)\left(\left(c_p P_t^p + c_{op}P_t^{op}\right)x - \kappa_i\right)dt$$
$$-K_i\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i}\hat{\mu}^i(dt,da,dx)e^{-(\rho+\gamma_i)t} + \widetilde{K}_i\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i}\mu^i(dt,da,dx)e^{-(\rho+\gamma_i)t}$$

Linear programming

MFG equations: constraints I

$$\int_{[0,T]\times A\times O_i} u(t,a,x)\nu_i(dt,da,dx) + \int_{[0,T]\times A\times O_i} \left(\frac{\partial u}{\partial t} + L_i u\right) m_i^t(da,dx)dt$$
$$= \int_{[0,T]\times A\times O_i} u(t,a,x)\mu_i(dt,da,dx)$$

$$\int_{[0,T]\times O_i} u(t,x)\hat{\nu}_i(dt,dx) + \int_{[0,T]\times O_i} \left(\frac{\partial \hat{u}}{\partial t} + \hat{L}_i u\right) \hat{m}_i{}^t(dx)dt$$
$$= \int_{[0,T]\times O_i} u(t,x)\hat{\mu}_i(dt,dx)$$

MFG equations: constraints II

$$\hat{\nu}_i(dt, dx) = \hat{\nu}_0^i(dx)\delta_0(dt) \tag{23}$$

$$\nu_i(dt, da, dx) = \nu_0^i(da, dx)\delta_0(dt) + \hat{\mu}_i(dt, dx)\delta_0(da) \tag{24}$$

▸ Linear programming

Nash equilibrium equations

- Denote $\mathcal{R}_i(\hat{\nu}_0^i, \nu_0^i)$ the class of n-uplets:

$$\left(\hat{\mu}^i, \left(\hat{m}_t^i\right)_{0\leq t\leq T}, \mu^i, \left(m_t^i\right)_{0\leq t\leq T}\right) \in \mathcal{M}_i \times \mathcal{V}_i \times \mathcal{M}_i \times \mathcal{V}_i \tag{25}$$

with for all $u \in C_b^{1,2,2}\left([0, T] \times \mathcal{A} \times \overline{\mathcal{O}}_i\right)$ satisfies

Nash equilibrium equations

The class satisfies the constraints:

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\nu^i(dt,da,dx) + \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \left\{\frac{\partial u}{\partial t} + \mathcal{L}_i u\right\} m_t^i(da,dx)dt$$

$$= \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\mu^i(dt,da,dx) \quad (11)$$

$$\int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\hat{\nu}^i(dt,da,dx) + \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} \left\{\frac{\partial u}{\partial t} + \mathcal{L}_i u\right\} \hat{m}_t^i(da,dx)dt$$

$$= \int_{[0,T]\times\mathcal{A}\times\overline{\mathcal{O}}_i} u(t,a,x)\hat{\mu}^i(dt,da,dx)$$

Nash equilibrium equations

$$\hat{\nu}_i(dt, dx) = \hat{\nu}_0^i(dx)\delta_0(dt) \tag{26}$$

$$\nu_i(dt, da, dx) = \nu_0^i(da, dx)\delta_0(dt) + \hat{\mu}_i(dt, dx)\delta_0(da) \tag{27}$$

▸ Nash equilibrium

Numerical resolution: the fictitious play algorithm

- For each group of technologies:
  1. Initialize with a "guess" on the strategy
  2. Describe optimal strategies for a representative agent as a function of the population distribution
  3. Population distribution update in case of strategy profitability
  4. Repeat until stationarity of the strategy (no more profitable deviation)
     $\rightarrow$ **Satisfactory approximation of Nash Equilibrium**

▸ Numerical algorithm

Numerical resolution : The fictitious play algorithm

1. Initialization:

$$\left(\hat{\mu}^{i,0}, \left(\hat{m}_t^{i,0}\right)_{0\leq t\leq T}, \mu^{i,0}, \left(m_t^{i,0}\right)_{0\leq t\leq T}\right) \in \mathcal{R}_i, \quad i = 1, \dots, N+\bar{N}$$

2. Compute prices $(P_{tp}, P_{top}, P_{t1}...P_{tK})_{0\leq t\leq T}$
3. Optimize the agents program to get best responses
4. Measures update:

$$\left(\hat{\mu}^{i,j}, \left(\hat{m}_t^{i,j}\right)_{0\leq t\leq T}, \mu^{i,j}, \left(m_t^{i,j}\right)_{0\leq t\leq T}\right) = \varepsilon_j \left(\hat{\bar{\mu}}^{i,j}, \left(\hat{\bar{m}}_t^{i,j}\right)_{0\leq t\leq T}, \bar{\mu}^{i,j}, \left(\bar{m}_t^{i,j}\right)_{0\leq t\leq T}\right) + (1-\varepsilon_j)\left(\hat{\mu}^{i,j-1}, \left(\hat{m}_t^{i,j-1}\right)_{0\leq t\leq T}, \mu^{i,j-1}, \left(m_t^{i,j-1}\right)_{0\leq t\leq T}\right)$$

▸ Linear programming