Systemic risk in energy derivative markets: a graph-theory analysis

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Objectives

• Empirical study on systemic risk in derivative markets
• Approach in three dimensions
  - Observation time
  - Spatial integration
  - Maturity of the transactions
• Influence of physical as well as derivative markets
• Integration as a necessary condition for systemic risk to appear
• Correlations, co-movements
Selected markets

- Choice directed by:
  - Concerns about speculation in commodities
  - Development of bio fuels
  - Portfolio management / Commodities as a new class of assets
  - Organized markets with the highest transaction volumes

- 14 markets (> 760 000 daily futures prices (settlement))
- 1998 - 2011
Methodology

- Huge volume of data + 3 dimensional analysis
- Price system = complex evolving system
- Use of methods originated from statistical physics
- Graph-theory and networks
- Graph:
  - Nodes: time series of daily futures returns
  - Links: correlations between the price returns
- Full connected graph:
  all possible connections between N nodes (time series of price returns) with \(\frac{N(N-1)}{2}\) links
Example of the crude oil market
(1 market, maturity dimension)
Methodology

- Information in the graph is filtered
  - most relevant links
  - highest correlations

- Minimum Spanning Trees (MST)
  Mantegna (1999)
  Path for the propagation of prices fluctuations

1. How did we get minimum spanning trees?
2. Our results with the MST, in the 3 dimensions:
   maturity, space, 3D
3. Dynamical analysis on the graphs and on the MST
1. Minimum spanning trees

- Synchronous correlation coefficients $\rho$ of prices returns $r$:

$$ r_i = \frac{(\ln F_i(t) - \ln F_i(t - \Delta t))}{\Delta t} $$

$$ \rho_{ij}(t) = \frac{\langle r_i r_j \rangle - \langle r_i \rangle \langle r_j \rangle}{\sqrt{\left(\langle r_i^2 \rangle - \langle r_i \rangle^2\right)\left(\langle r_j^2 \rangle - \langle r_j \rangle^2\right)}} $$

- With: F(t), futures prices at t
- Correlation matrix C, (NxN), symmetric
From correlations to distances

• Non linear transformation
• Distances $d$ between two nodes defined as follows:

$$d_{ij} = \sqrt{2(1 - \rho_{ij})}$$

• Distance matrix D, (NxN)
• Full connected graph
  - represents all the possible connections between N nodes
  - weighted by the distances
Minimum spanning tree

- All the nodes of the graph are spanned
- No loops
- Result: links of the MST are a subset of the initial graph
- The information space is reduced from \( \frac{N(N-1)}{2} \) to \( (N-1) \)
- In this study: shortest path linking all nodes
  
  Easiest path for the transmission of prices shocks
2. Topology of the MST

II) Methodology: Graphs and Minimum Spanning Trees (MST)

Figure: Star-like and chain-like trees

Lautier, Ling, Raynaud
Systemic risk in commodity markets
AFFI May 2014 5 / 22
2. Topology of the MST

2.1. Maturity dimension

Heating oil – Month 1 to 18

Samuelson effect

15/05/2014 Dauphine
Evolution of the integration through time: Eurodollar

1998-2001
Evolution of the integration through time: Eurodollar

1998-2001

2001-2004

15/05/2014 Dauphine 14
Evolution of the integration through time: Eurodollar

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2004-2009

15/05/2014
Evolution of the integration through time, US natural gas

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Evolution of the integration through time, US natural gas

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2004-2009
2.2. Spatial dimension

- Wheat
- Corn
- Soy Bean
- Soy Oil
- Crude, US
- Crude, UK
- Heating oil, US
- S&P 500
- Nat. Gas, US
- Nat. Gas, UK
- Gas oil, UK
- Gold
- Exchange rates
- Interest rates
- Crude, US
- Crude, UK
- Heating oil, US
- S&P 500
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2.2. Spatial dimension

Agriculture

Wheat → Corn → Soy Bean → Soy Oil

Interest rates

Exchange rates

Gold

Crude, US

Crude, UK

Heating oil, US

Gas oil, UK

Nat. Gas, US

S&P 500

Nat. Gas, UK
2.2. Spatial dimension

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Energy
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2.4 Allometric coefficients

- Quantifying the degree of linearity in the trees
- The root is the node with the highest connectivity
- Starting from this root, two coefficients $A_i$ and $B_i$ are assigned to each node $i$:

\[
A_i = \sum_j A_j + 1 \\
B_i = \sum_j B_j + A_i \\
B \sim A^\eta
\]

Where $\eta$ is the allometric exponent

$\eta$ stands between 1+ (star-like) and 2- (chain-like)
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3. Dynamical studies

3.1. Full connected graph: mean correlations and their variances (3-D)
3.2. Node’s strength

- Full connected graph
- The node’s strength $S_i$ indicates the closeness of one node $i$ to all others:

$$S_i = \sum_{i \neq j} \frac{1}{d_{ij}}$$
3.3. Normalized tree’s length

- Sum of the lengths of the links belonging to the MST:

\[
L(t) = \frac{1}{N - 1} \sum_{(i,j) \in MST} d_{ij}
\]

- The more the length shortens, the more integrated the system is
3.4. Survival ratios

- Robustness of the topology over time
- The survival ratio $S_R$ refers to the fractions of edges in the MST, that survives between two consecutive trading days:

$$S_R(t) = \frac{1}{N-1} |E(t) \cap E(t-1)|$$

$E(t)$ : set of edges at $t$
Pruning the trees

- Analysis of inter-market and inter-sectors reorganizations
- Consider only the links between markets, whatever the maturity is considered
Pathological configuration: an example
Pathological configuration: an example

Energy

Finance

Agriculture
Most stable links

(a) SoyOil-SoyBean SoyBean-Corn Wheat-Corn

(b) GasNatEu-Gold

Eurodol-Gasoil Eurodol-Gold

Brent crude-SoyBean NatGasUS-SoyOil Heating oil-SoyBean

Gold-FxRate Brent crude-Light crude Eurodol-Exrate

Light crude-Heating oil Gasoil-Heating oil
Main results - Extensions

MAIN RESULTS

1. Topology
   - Chain-like trees in the maturity dimension
   - Star-like trees in the spatial and 3-D dimensions

2. Emerging taxonomy
   - Trees organized around the three sectors of activity
   - Center of the graph: two crude oils

3. Integration
   - Increases in all dimensions (spatial, maturity, 3D)
   - Progresses at the heart of the system

EXTENSIONS

1. Introducing directions in the graph
2. Event studies / financial crises
1. Introducing directions in the graph

• Full connected graph
• Information flows:
  - static analysis
  - dynamic analysis
Full connected graph, maturity dimension
Information flows: static analysis

Figure: Average information transfer between maturities, 2001-2011
Figure: Information transfer between maturities, 2001-2011
2. Event studies / financial crises

Centrality measures

![Diagram of network with nodes labeled E, F, and D, illustrating the structure of the path of propagation.](image)